GRADE 12

Analytical Geometry

WEBSITE NOTES

TOPIC: The equation of a circle (any centre)

$$(x-a)^2+(y-b)^2=r^2$$

Explanation:

- $(x)^2+(y)^2=r^2$ If the centre of the circle is at the origin (0;0). r is the radius, given or you will have to work it out in an application question.

- $(x-a)^2+(y-b)^2=r^2$ If the centre of the circle is at the point (a; b). r is the radius, given or you will have to work it out in an application question.

Example 1

Work out the equation of the circle at the origin with a radius of 5.

Answer

$(x)^2+(y)^2=r^2$

$(x)^2+(y)^2=5^2$

$(x)^2+(y)^2=25$

Example 2

Work out the equation of the circle centre at the point (3; 2) with a radius of 5.

Answer

Centre is now (3;2) which means that a = 3 and b = 2

$(x-a)^2+(y-b)^2=r^2$

$(x-3)^2+(y-2)^2=5^2$

$(x-3)^2+(y-2)^2=25$

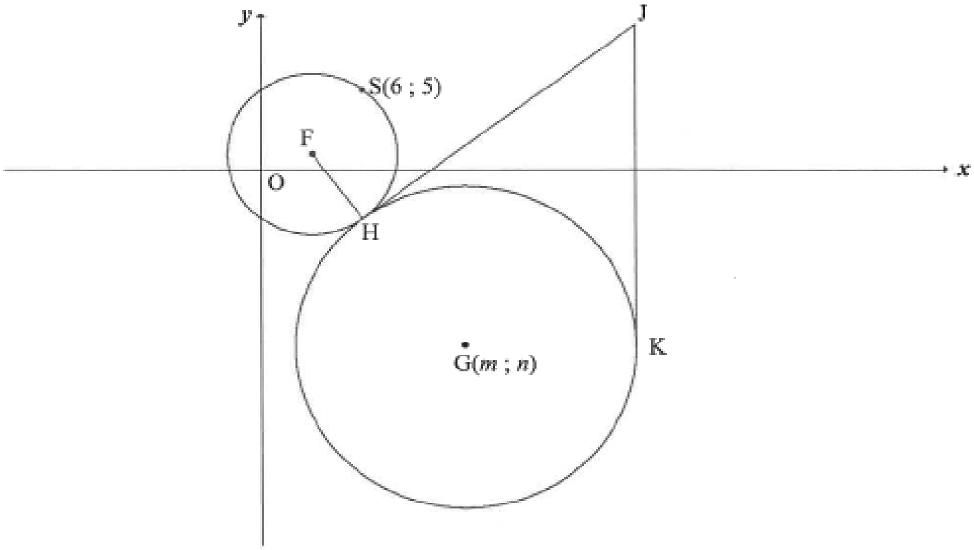
Example 3 part 1

Past Paper Example

2018 November Gr 12 Paper 2 (Broken into different parts to try understand better)

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G(m ; n) in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

Answer

4.1 F is the centre of the Circle F.

The formula as given in the question $(x-3)^2+(y-1)^2=r^2$ (no radius asked for yet)

Therefore, the coordinates of F is (3; 1)

4.2 FS is a length of a line to work out. It will also represent the radius because it is from the centre of circle to circle circumference.
To work out the length, the distance formula will be used.

$FS = \sqrt{(x-a)^2+(y-b)^2}$
x will be 6
y will be 5
a will be 3
b will be 1

$$FS = \sqrt{(6-3)^2+(5-1)^2}$$
$$FS = 5$$

Therefore the radius is 5. Remember this for possible further questions. There may be or may not be questions that require you to use the radius of this circle.

Example 3 part 2

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

Answer

4.3 ***The ratio of FH: HG is 1:2 (Does not mean FH is 1 and HG is 2)***
FH is the radius of the circle F. The radius is 5 (worked out in 4.2)
Write it in a fraction form

$$\frac{FH}{HG} = \frac{1}{2}$$

Cross multiply

$$2FH = HG$$

Substitute the radius in place of FH

$$2 \text{ x } (5) = \text{HG}$$

HG = 10

4.4
Tangents from common/same point /

Example 3 part 3

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

Answer
4.5.1
Geometry and analytical geometry used together.

$F\hat{H}J = 90°$ [tan ⊥ radius / *rkl ⊥ radius*]
$FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*]
FJ = $\sqrt{425}$ or $5\sqrt{17}$ or 20,62

4.5.2
Using the formula
$Radius^2 = \sqrt{(x-a)^2 + (y-b)^2}$
a = m
b = n
HG is the radius and = 10 from Question 4.3

$100 = \sqrt{(x-m)^2 + (y-n)^2}$

4.5.3

K(22; n) [radius ⊥ tangent]
GK = HG = 10 [radii]
FH = FS = 5 [radii]
$m = 22 - 10$
$m = 12$
F, H and G are collinear [HJ is a common tangent]
F, H en G is saamlynig *[HJ is 'n gemeemskaplike raaklyn]*
$FG^2 = (12-3)^2 + (n-1)^2$
$15^2 = 81 + (n-1)^2$
$(n-1)^2 = 144$
$n - 1 = \pm 12$ **OR/*OF***
$n \neq 13$ or $n = -11$
$\therefore G(12; -11)$

$n^2 - 2n - 143 = 0$
$(n+11)(n-13) = 0$
$n = -11$ or $n \neq 13$

Collinear means on the same line.

The x coordinate at K is 22 because the equation of JK is x = 22(given).

At m, the x-coordinate of G, is the value now 10 less because the radius of the circle is 10.

The answer can not be n = 13 because G is where x is positive, and y is negative.

Example 4
Determine the value of g if (3; $\sqrt{g}$) is a point on the circle $x^2+y^2 = 19$.

Answer
$x^2+y^2 = 19$.
$(3)^2+(\sqrt{g})^2 = 19$.
9+g=19
g = 10