

GRADE 12

Analytical Geometry

WEBSITE NOTES 2

TOPIC: The equation of a circle (any centre)

$$(x - a)^2 + (y - b)^2 = r^2$$

In Grade 11 you learnt:

1. Distance Formula

$$AB = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

2. Gradient between two points

$$m_{AB} = \frac{y_a - y_b}{x_a - x_b}$$

3. The MIDPOINT between two points

$$\text{Midpoint AB} = \left(\frac{x_a + x_b}{2}, \frac{y_a + y_b}{2} \right)$$

4. $m = \tan A$ (where m is the gradient of a line and A is the angle of inclination)

Examples to try

Determine the equation of the circle with:

1. The centre is the origin and passing through the point (2;3)
2. The centre is (-2;4) and radius is 8.
3. Diameter CD where C = (1; -4) and D = (-3; 9).

Answers

1. $(2)^2 + (3)^2 = r^2$

$$4 + 9 = r^2$$

$$13 = r^2$$

$$(x)^2 + (y)^2 = 13$$

2. $(x + 2)^2 + (y - 4)^2 = 64$

3. $CD = \sqrt{(x_c - x_d)^2 + (y_c - y_d)^2}$

$$CD = \sqrt{(1 + 3)^2 + (-4 - 9)^2}$$

$$CD = \sqrt{(4)^2 + (-13)^2}$$

$$CD = \sqrt{16 + 169}$$

$$CD = \sqrt{185}$$

Therefore the radius will be $\frac{1}{2} \times \sqrt{185}$

The centre of the circle is obtained by using the midpoint formula

$$= \left(\frac{x_c + x_d}{2}, \frac{y_c + y_d}{2} \right)$$

$$= \left(\frac{1 + (-3)}{2}, \frac{-4 + 9}{2} \right)$$

$$= \left(\frac{-2}{2}, \frac{5}{2} \right)$$

$$= \left(-1; \frac{5}{2} \right)$$

Therefore, the formula of the circle is

$$(x + 1)^2 + \left(y - \frac{5}{2} \right)^2 = \left(\frac{\sqrt{185}}{2} \right)^2$$

$$(x + 1)^2 + \left(y - \frac{5}{2} \right)^2 = \frac{185}{4}$$

Website Exercise 1

Exercise 1

- 1.1 Yes. Centre is the origin.
- 1.2 Yes. Centre is the origin.
- 1.3 No. $-2y^2$

2.1 $x^2 + y^2 = 49$

2.2 Sub the point (8;-3) into $x^2 + y^2 = r^2$

$$(8)^2 + (-3)^2 = 64 + 9 = 73$$

$$r^2 = 73$$

$$x^2 + y^2 = 73$$

2.4 $(x - 1)^2 + (y - 3)^2 = 37$ (similar to Example no 3 above)

2.6 $(x - 1)^2 + (y - 1)^2 = 5$ or $(x - 3)^2 + (y - 1)^2 = 5$ (similar to Example no 3 above but do not have to work out the midpoint because the points are not the diameter.

You will substitute the points into the circle formula and obtain a and b using simultaneous equations

Substitute (2;3) and $r = \sqrt{5}$ into the circle formula to obtain equation 1

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(2 - a)^2 + (3 - b)^2 = \sqrt{5}^2$$

Substitute (2;-1) and $r = \sqrt{5}$ into the circle formula to obtain equation 2

$$(2 - a)^2 + (-1 - b)^2 = \sqrt{5}^2$$

SOLVE SIMULTANEOUSLY

3.1 radius = 11 and centre is origin

3.2 radius is $\sqrt{3}$ and centre is origin. Divide everything first by 3 and move -9 to the right hand side of equal sign.

3.3 Centre is (1;5) and radius is 4