GRADE 12

Analytical Geometry

WEBSITE NOTES 3

TOPIC: The equation of a circle (any centre)

$(x-a)^2+(y-b)^2=r^2$

The equation of a tangent to a circle

In Grade 11 you learnt:

1. Distance Formula
   $AB=\sqrt{(x_a-x_b)^2+(y_a-y_b)^2}$
2. Gradient between two points
   $mAB=\frac{y_a-y_b}{x_a-x_b}$
3. The MIDPOINT between two points
   Midpoint AB $=(\frac{x_a+x_b}{2};\frac{y_a+y_b}{2})$
4. m = tan A (where m is the gradient of a line and A is the angle of inclination)

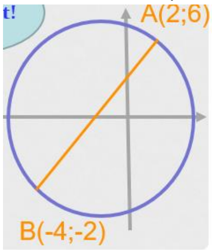
Revise to work out the equation of a circle

Diameter is 2 x radius

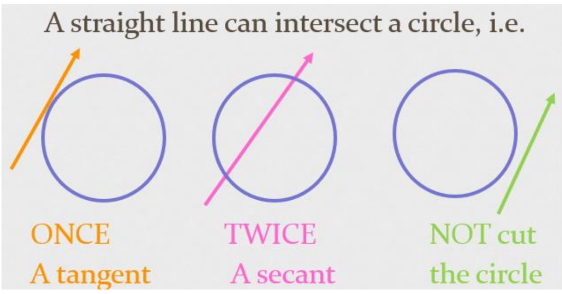
Examples to try

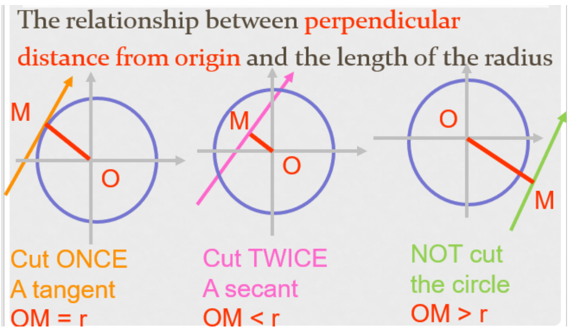
1. ***Example where you are given the centre and the radius***
   Find the general equation a circle centre (-1;3) with a radius of 5 units.

2. ***Example where you are given the Diameter***
   Determine the equation the circle, given the diameter AB



Tangents





Where r is the radius

Example 1

Show that 2x + 5 + y = 0 is a tangent to the circle $x^2 + y^2 = 5$.

You will need to use simultaneous equations in this type of example. WHY???

You need to show that there is only 1 point of intersection. So you need to work out for x or for y. Look at the equations given and decide on the easier one to work out.

In this example 2x + 5 + y = 0 can be easily changed to y = - 2x – 5 and then substituted into $x^2 + y^2 = 5$.

y = - 2x – 5 ……… (1) $x^2 + y^2 = 5$ ……….. (2)

Substitute (1) into (2)

$x^2 + (-2x - 5)^2 = 5$

We are going to solve then only for x. If the x-values are the same, then the y-values will also be the same if we substitute back.

$x^2 + (-2x - 5)^2 = 5$

$x^2 + (4x^2 + 20x + 25) = 5$

$5x^2 + 20x + 20 = 0$

$x^2 + 4x + 4 = 0$

$(x + 2)(x + 2) = 0$

x = -2 or x = -2

Since there is only 1 point of intersection (i.e. x= -2), the line is a TANGENT

If 2 DIFFERENT points of intersection are found, the line is a SECANT.

If there is no solution, the line does not intersect the circle.

Example 2

Find the equation of the tangent to the circle $x^2 + y^2 = 5$ at the point (– 2; 1).

REMEMBER:

Radius is ⊥ to Tangent

$m_{radius} \times m_{tangent} = -1$

m being the gradient

Answer

Tangent: y = mx + c

Tangent is a straight line, so we use the equation of a straight line

$m_{radius} = \frac{1-0}{-2-0} = \frac{-1}{2}$

Work out the gradient of the radius. The centre of the circle is (0;0).

Radius is ⊥ to Tangent

$m_{radius} \times m_{tangent} = -1$

$\therefore m_{tangent} = 2$

$y = 2x + c$

Solve for c now.

Substitute point of contact of tangent:

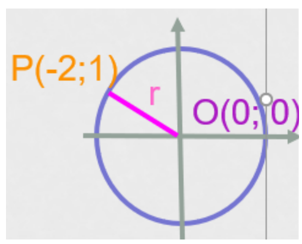
Subst. P(-2; 1)

$1 = 2(-2) + c$

$c = 5$

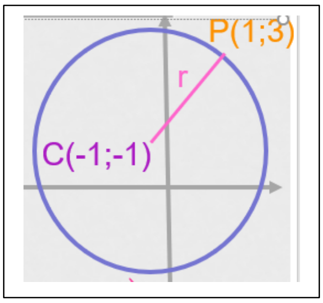
$\therefore$ *Equation of tangent is:*

$y = 2x + 5$



Example 3 (Try yourself)

Find the equation of the tangent at the point P(1;3), given the centre C(-1;-1).



Example 4 (Try yourself)

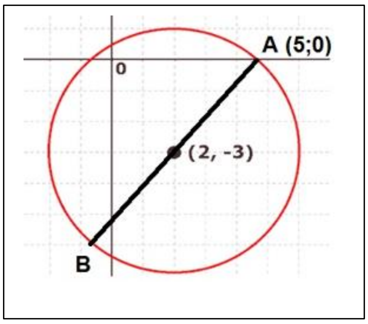
Find the equation of the tangent to the circle $(x-2)^2 + (y-3)^2 = 16$ at (6;0).

HINT: the centre of the circle you can get from the circle formula.

Example 5 (Try yourself)

Given the circle with diameter AB and centre (1;-1).

1. Find the equation of the circle.



Work out the co-ordinates of B using midpoint theorem

2. The equation of the tangent at A
3. Determine the equation of a line parallel to the tangent at A and passing through the point C (2;8).
4. Determine whether the tangent at B will intersect with the tangent at A.

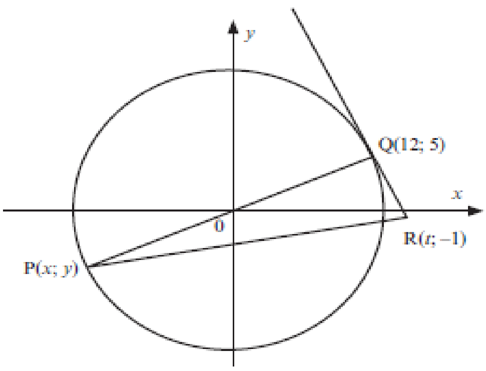
   ***HINT:* Work out the equation of the Tangent at B and equate Tangent at A to Tangent at B.**

Example 6 (Try yourself)

Find the equation of the tangent APB which touches a circle centre C with equation $(x-3)^2 + (y+1)^2 = 20$ at P(5; 3).

Example 7 (Try yourself)

2. O is the centre of the circle in the figure below. P(x; y) and Q(12; 5) are two points on the circle. POQ is a straight line. The point R(t; –1) lies on the tangent to the circle at Q.



2.1	Determine the equation of the circle.	(3)
2.2	Determine the equation of the straight line through P and Q.	(2)
2.3	Determine x and y, the coordinates of P.	(4)
2.4	Show that the gradient of QR is $-\frac{12}{5}$.	(2)
2.5	Determine the equation of the tangent QR in the form $y = \ldots$	(3)
2.6	Calculate the value of t.	(2)
2.7	Determine an equation of the circle with centre Q(12; 5) and passing through the origin.	(3)
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