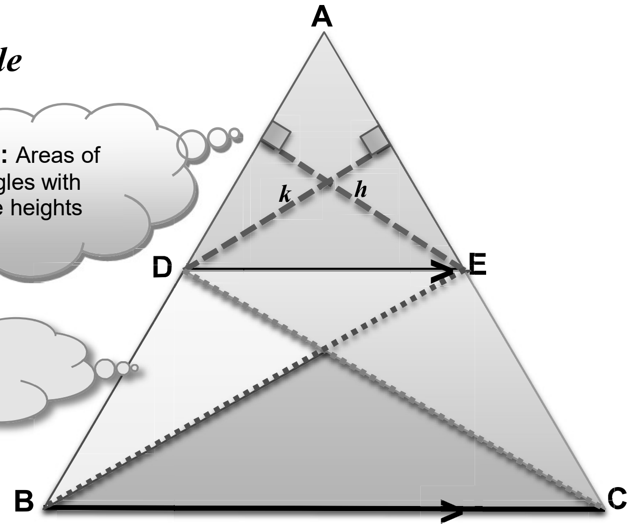
Theorem 5

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

line || one side of Δ or prop theorem; name ||



Hint 1: Areas of triangles with same heights

Given: ΔABC with DE||BC

Hint 2: Δ^s on same base.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: In ΔADE, draw height h relative to basis AD and the height k relative to basis AE. Combine BE and DC to form ΔBDE and Δ CED.

Proof:

$$\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE} = \frac{\frac{1}{2} \times AD \times h}{\frac{1}{2} \times BD \times h} = \frac{AD}{BD}$$

$$\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta CED} = \frac{\frac{1}{2} \times AE \times k}{\frac{1}{2} \times EC \times k} = \frac{AE}{EC}$$

Hint 3: Compare the areas

$$\therefore Area\ \ \Delta BDE = Area\ \ \Delta CED$$

$(Same\ base,\ same\ height\ and\ lies\ between\ parallel\ lines)$

$$\therefore \frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE} = \frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta CED}$$

$$\therefore \frac{AD}{BD} = \frac{AE}{EC}$$

DEDUCTIONS

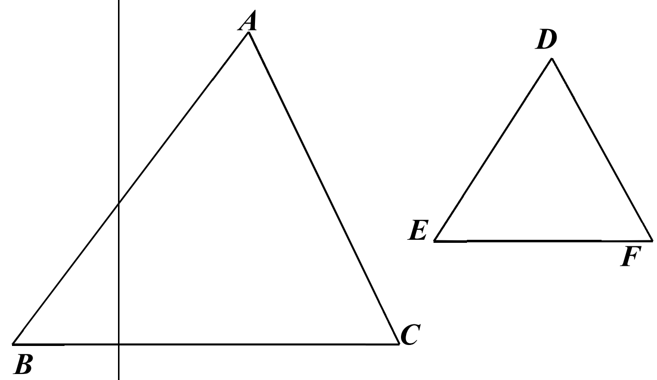
$$\frac{AB}{AD} = \frac{AC}{AE}; \quad \frac{AB}{DB} = \frac{AC}{EC};$$

$$\frac{AD}{DB} = \frac{AE}{EC}; \quad \frac{DB}{AB} = \frac{AC}{EC}$$

Theorem 6

If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).

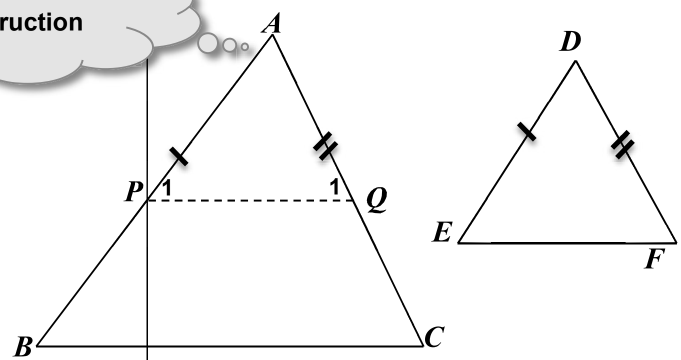
||| Δ^s or equiangular Δs



Given: Δ ABC and Δ DEF *with*

$\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ *and* $\hat{C} = \hat{F}$

Construction



To Prove: $$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

Construction
On AB, mark off P and on AC mark off Q such that AP = DE and AQ = DF. Draw PQ.

Proof:

1. CONGRUENCY

In Δ APQ and Δ DEF:

AP = DE [Construction]

$\hat{A} = \hat{D}$ [Given]

AQ = DF [Construction]

$\therefore$ Δ APQ $\equiv$ Δ DEF [s; $\angle$; s]

2. PARALLEL LINES

$\hat{P}_1 = \hat{E}$ [From congruency]

$\hat{B} = \hat{E}$ [Given]

$\hat{P}_1 = \hat{B}$ [both = $\hat{E}$]

$\therefore$ PQ // BC [Corresp $\angle^s$ equal]

3. Use THEOREM 1

$\frac{AP}{AB} = \frac{AQ}{AC}$ [line || one side of Δ ; PQ // BC in Δ ABC]

But AP = DE and AQ = DF [Constr]

$$\frac{DE}{AB} = \frac{DF}{AC}$$

4. New CONSTRUCTION

Similarly by marking off X and Y on AB and BC, we can prove that: $\frac{DE}{AB} = \frac{EF}{BC}$

$$\therefore \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$