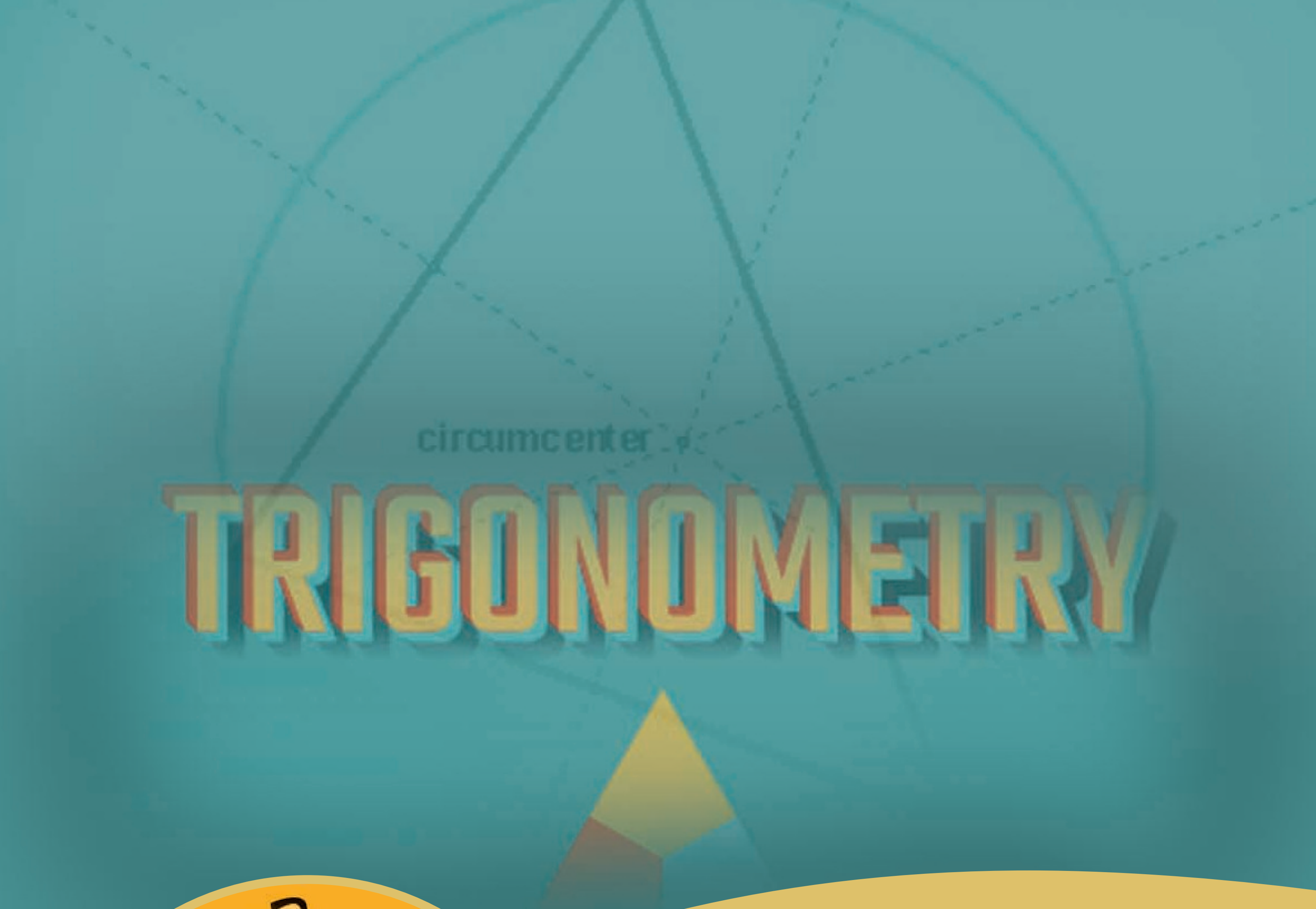

MATHEMATICS

TRIGONOMETRY

GRADE 12

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Read to Lead
A Reading Nation is a Leading Nation

2030 NDP

Foreword

In order to improve learning outcomes the Department of Basic Education conducted research to determine the specific areas that learners struggle with in Grade 12 examinations. The research included a trend analysis by subject experts of learner performance over a period of five years as well as learner examination scripts in order to diagnose deficiencies or misconceptions in particular content areas. In addition, expert teachers were interviewed to determine the best practicesto ensure mastery of thetopic by learners and improve outcomes in terms of quality and quantity.

The results of the research formed the foundation and guiding principles for the development of the booklets. In each identified subject, key content areas were identified for the development of material that will significantly improve learner's conceptual understanding whilst leading to improved performance in the subject.

The booklets are developed as part of a series of booklets, with each bookletfocussing onlyon one specific challenging topic. The selected content is explained in detail and include relevant concepts from Grades 10 - 12 to ensure conceptual understanding.

The main purpose of these booklets is to assist learners to master the content starting from a basic conceptual level of understanding to the more advanced level. The content in each booklets is presented in an easy to understand manner including the use of mind maps, summaries and exercises to support understanding and conceptual progression. These booklets should ideally be used as part of a focussed revision or enrichment program by learners after the topics have been taught in class. The booklets encourage learners to take ownership of their own learning and focus on developing and mastery critical content and skills such as reading and higher order thinking skills.

Teachers are also encouraged to infuse the content into existing lesson preparation to ensure in-depth curriculum coverage of a particular topic. Due to the nature of the booklets covering only one topic, teachers are encouraged to ensure learners access to the booklets in either print or digital form if a particular topic is taught.

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2. How to use this booklet

This booklet is designed to clarify the content prescribed in Mathematics. In addition, it has some tips on how you should tackle real-life problems on a daily basis.

Candidates will be expected to have already mastered the content outlined for Grades 8-11. This booklet must be used to master some mathematical rules that you may not yet be aware of. The prescribed textbook must also be used.

3. Study and examination tips

All learners should be able to acquire sufficient understanding and knowledge to:

- develop fluency in computation skills without relying on the use of a calculator;
- generalise, make conjectures and try to justify or prove them;
- develop problem-solving and cognitive skills;
- make use of the language of Mathematics;
- identify, investigate and solve problems creatively and critically;
- use the properties of shapes and objects to identify, investigate and solve problems creatively and critically;
- encourage appropriate communication by using descriptions in words, graphs, symbols, tables and diagrams; .
- practise Mathematics every day.

4. Overview of trigonometry

TRIGONOMETRY

Trigonometric ratios

$\sin A = \dfrac{opposite}{hypotenuse}$

$\cos A = \dfrac{adjacent}{hypotenuse}$

$\tan A = \dfrac{opposite}{adjacent}$

Trigonometric identities

$\sin^2 x + \cos^2 x = 1$

$\cos 2x = \cos^2 x - \sin^2 x$ *or*

$= 2\cos^2 x - 1$ *or*

$= 1 - 2\sin^2 x$

$\sin 2x = 2 \sin x \cos x$

2D and 3D problems

Trigonometric graphs

Triangle identities

Sine rule: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Cosine rule:

$c^2 = a^2 + b^2 - 2ab\cos C$

Area rule: area of

$Area\ of\ \Delta ABC = \dfrac{1}{2} ab \sin C$

$or\ \dfrac{1}{2} base \times \perp height$

Angles

Negative angles

Special angles

Compound and double angles

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

Trigonometry was developed in ancient civilisations to solve practical problems, such as those encountered in building construction and when navigating by the stars. We will show that trigonometry can also be used to solve some other practical problems. We use trigonometric functions to solve problems in two and three dimensions that involve right-angled triangles and non-right-angled triangles.

Trigonometric ratios, identities and reduction

Definitions: The trigonometric ratios are for right-angled triangles. These ratios all involve one angle (other than the right angle) and the length of two sides. The ratios can be used to find the length of an unknown side or an angle if the other two quantities are known.

The Pythagoras theorem states that for any right-angled triangle, the square on the hypotenuse is equal to the sum of the square of the other two sides.

The converse of this theorem states that if the square on the longest side of the triangle is equal to the sum of the square of the other two sides, then the triangle is a right-angled triangle.

Pythagoras: $AB^2 + BC^2 = AC^2$

Hints for solving two-dimensional problems using trigonometry and the Pythagoras theorem.

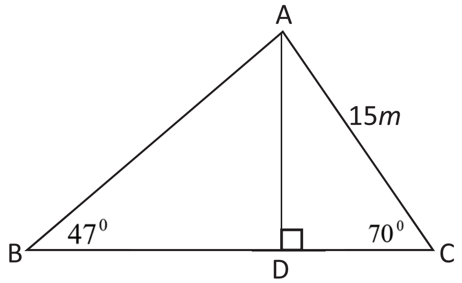
- If you are not given a diagram, draw one yourself.
- Mark all right angles on the diagram and fill in the figures for any other angles and lengths that are known.
- Mark the angles or sides that you have to find.
- Identify the right-angled triangles that you can use to find the missing angles or sides.
  - ✓ Decide what mathematical method you will use: Pythagoras, sin, cos or tan.
- Later in the problem, if you have to use a value that you have calculated, use the most accurate value and only round off at the end.

Example: Trigonometric ratios

Use the sketch below:

1. Write down the trigonometric ratios of angle B and angle C.
2. Solve for BD and AB.

Solutions:



1. $\sin B = \frac{AD}{AB}$; $\cos B = \frac{BD}{AB}$ and $\tan B = \frac{AD}{BD}$

$\sin C = \frac{AD}{AC}$; $\cos C = \frac{DC}{AC}$ and $\tan C = \frac{AD}{DC}$

2. $\sin 70^0 = \frac{AD}{AC}$

$AD = 15 \times \sin 70^0 = 14{,}095$

$\sin 47^0 = \frac{AD}{AB} = \frac{15 \times \sin 70^0}{AB}$

$AB = \frac{15 \times \sin 70^0}{\sin 47^0} = 19.27$ and $\cos 47^0 = \frac{BD}{AB} = \frac{BD}{19{,}27}$

$BD = 19{,}27 \times \cos 47^0 = 13{,}14$

Identities

Prior knowledge:

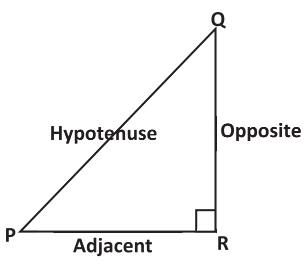
- Trigonometric ratios
- Pythagoras theorem
- Basic arithmetic involving fractions
- Basic algebra and factorisation

Revise the following trigonometric ratios for right-angled triangles that you learnt in Grade 10:

✓ $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

✓ $\cos\theta = \frac{\text{Adjecent}}{\text{Hypotenuse}}$

✓ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



$PR^2 + QR^2 = PQ^2$ ....[Phythagoras theorem]

Identities: definition of concepts/ theory

- A trigonometric identity is a mathematical statement that is true for all values of the angle, except those for which the statement is not defined.
- Identities are used in order to simplify trigonometric expressions.
- In Grade 11, there are two fundamental identities that you are required to know:
  - ✓ $\sin^2 A + \cos^2 A = 1$
  - ✓ $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- Identities have to be used in most cases to simplify expressions or to prove that the left-hand side of an identity is equal to the right-hand side of the identity.
- Tips when solving problems using identities:
  - ✓ Write expressions in terms of $\sin\theta$ and $\cos\theta$ only, by using $\tan\theta = \frac{\sin\theta}{\cos\theta}$
  - ✓ Note that: $\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} \rightarrow 1 \div \tan\theta = \frac{1}{1} \div \frac{\sin\theta}{\cos\theta} = \frac{1}{1} \times \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$
  - ✓ When dealing with fractions, find an LCD and add.
  - Factorise expressions where possible.

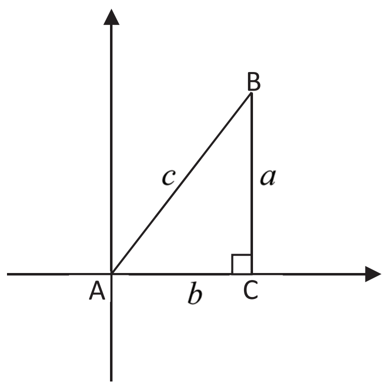
Prove the identities of: $\sin^2 A + \cos^2 A = 1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

To prove: $\sin^2 A + \cos^2 A =$ **1**

In triangle ABC, if angle A is placed at the origin:

- $\sin A = \frac{opp}{hyp} = \frac{a}{c}$
- $\cos A = \frac{adj}{hyp} = \frac{b}{c}$

$$\therefore \sin^2 A + \cos^2 A = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2 + b^2}{c^2}$$
$$= \frac{c^2}{c^2}$$
$$= 1$$



But in any right-angled triangle

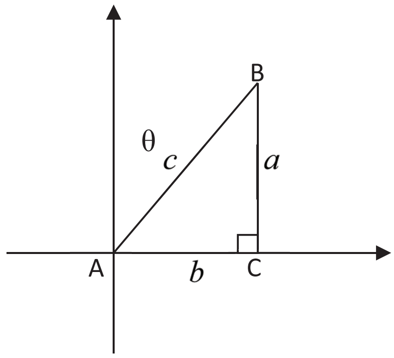
$\therefore a^2 + b^2 = c^2$ .... Pythagoras.

Identity: $\sin^2 A + \cos^2 A = 1$ can also be written as $\sin^2 A = 1 - \cos^2 A$ **or**

$\cos^2 A = 1 - \sin^2 A$

This identity is known as a square identity.

To prove that $\tan\theta = \frac{\sin\theta}{\cos\theta}$



RHS = $\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{a}{c}\right)}{\left(\frac{b}{c}\right)}$

$= \frac{a}{b}$

$= \tan\theta$

Example 1: Express $\frac{\sin\theta}{\tan\theta}$ as a single trigonometric ratio.

Solution: $\frac{\sin\theta}{\tan\theta}$

$= \sin\theta \div \frac{\sin\theta}{\cos\theta}$

$= \sin\theta \times \frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

Example 2: Prove the following identity: $\frac{(1-\sin\beta)(1+\sin\beta)}{\cos^2\beta} = 1$

Solution:

LHS = $\frac{(1-\sin\beta)(1+\sin\beta)}{\cos^2\beta}$

= $\frac{1-\sin^2\beta}{\cos^2\beta}$ $\rightarrow$ multiply the two brackets together

= $\frac{\cos^2\beta}{\cos^2\beta}$ $\rightarrow$ since $1-\sin^2\beta = \cos^2\beta$

= 1

$\therefore$ **LHS = RHS**

Example 3: Prove the following identity: $\frac{\sin\alpha - \sin\alpha\cos\alpha}{\cos\alpha - (1-\sin^2\alpha)} = \tan^2\alpha$

Solution:

LHS = $\frac{\sin\alpha - \sin\alpha\cos\alpha}{\cos\alpha - (1-\sin^2\alpha)} = \tan^2\alpha$

= $\frac{\sin\alpha - \sin\alpha\cos\alpha}{\cos\alpha - \cos^2\alpha}$ $\rightarrow \cos^2\alpha = 1 - \sin^2\alpha$

= $\frac{\sin\alpha(1-\cos\alpha)}{\cos\alpha(1-\cos\alpha)}$ $\rightarrow$ Factorise to simplify

= $\frac{\sin\alpha}{\cos\alpha}$ $\rightarrow$ Divide like factors

= $\tan\alpha$

Activities

1. Simplify the following expressions:

   1.1 $\tan x.\cos x$

   1.2 $\frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta}$

   1.3 $\frac{1}{\cos^2\theta} - 1$

2. Prove the following: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{2}{\cos^2\theta}$

3. Complete: $(1-\sin^2\theta) = (1-\sin\theta)(.............)$. Hence, prove the identity:

   $$\frac{1-\sin^2 A}{\cos A\left(\frac{1}{\cos A} + \tan A\right)} = 1 - \sin A$$

Solutions:

1.1	$\tan x.\cos x$ $= \frac{\sin x}{\cos x} \times \cos x$ $= \sin x$	1.2	$\frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta}$ $= \frac{1-\cos^2\theta}{\sin\theta}$ $= \frac{\sin^2\theta}{\sin\theta}$ $= \sin\theta$

1.3	$\frac{1}{\cos^2\theta}-1$ $=\frac{1-\cos^2\theta}{\cos^2\theta}$ $=\frac{\sin^2\theta}{\cos^2\theta}$ $=\tan^2\theta$	2	LHS = $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ = $\frac{1\times(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}+\frac{1\times(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$ = $\frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$ = $\frac{2}{\cos^2\theta}$
3	$(1-\sin^2\theta)=(1-\sin\theta)(1+\sin\theta)$ LHS = $\frac{1-\sin^2 A}{\cos A\left(\frac{1}{\cos A}+\tan A\right)}$ = $\frac{1-\sin^2 A}{\cos A\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)}$ = $\frac{(1-\sin A)(1+\sin A)}{1+\sin A}$ = $1-\sin A$ = RHS		

Exercises

1. Simplify the following expressions:

   1.1 $\tan x.\cos x$

   1.2 $\frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta}$

   1.3 $\frac{1}{\cos^2\theta}-1$

2. Prove the following: $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=\frac{2}{\cos^2\theta}$

3. Simplify the following expressions:

   1.1 $\tan x.\cos x$

   1.2 $\frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta}$

Reduction

Formulae

Derive and use reduction formulae to simplify the following expressions:

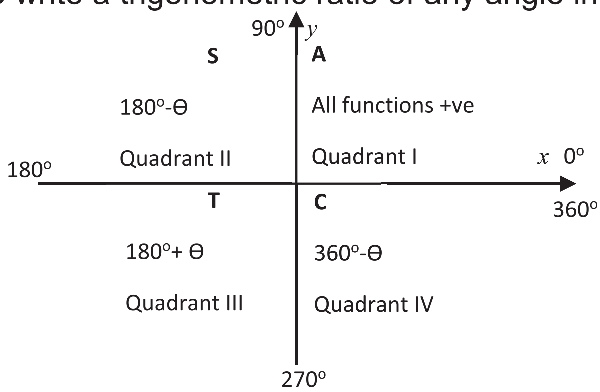
- ✓ $\sin(90^\circ \pm \theta), \cos(90^\circ \pm \theta)$
- ✓ $\sin(180^\circ \pm \theta), \cos(180^\circ \pm \theta), \tan(180^\circ \pm \theta)$
- ✓ $\sin(360^\circ \pm \theta), \cos(360^\circ \pm \theta), \tan(360^\circ \pm \theta)$
- ✓ $\sin(-\theta), \cos(-\theta), \tan(-\theta)$

Prior knowledge:

- Trigonometric ratios.
- Derive and use ratios (without using a calculator or special angles).
- Solve problems in the Cartesian plane.

Definition of concepts/ theory

Reduction formulae enable us to write a trigonometric ratio of any angle in terms of acute angle: (angles less than 90º).



Quadrant II	**Quadrant I**
$\sin(180^\circ - \theta) = +\sin\theta$ $\cos(180^\circ - \theta) = -\cos\theta$ $\tan(180^\circ - \theta) = -\tan\theta$	Reduction formulae are not used on angles less than 90º.
Quadrant III	**Quadrant IV**
$\sin(180^\circ + \theta) = -\sin\theta$ $\cos(180^\circ + \theta) = -\cos\theta$ $\tan(180^\circ + \theta) = +\tan\theta$	$\sin(360^\circ - \theta) = -\sin\theta$ $\cos(360^\circ - \theta) = +\cos\theta$ $\tan(360^\circ - \theta) = -\tan\theta$

Negative angles

Quadrant II	Quadrant III	Quadrant IV
$\sin(-180° - \theta) = +\sin\theta$ $\cos(-180° - \theta) = -\cos\theta$ $\tan(-180° - \theta) = -\tan\theta$	$\sin(-180° + \theta) = -\sin\theta$ $\cos(-180° + \theta) = -\cos\theta$ $\tan(-180° + \theta) = +\tan\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = +\cos\theta$ $\tan(-\theta) = -\tan\theta$

- NB: Adding or subtracting $360°$ does not change the value of a trigonometric function.
  - ✓ $\sin(\theta \pm 360°) = +\sin\theta$
  - ✓ $\cos(\theta \pm 360°) = +\cos\theta$
  - ✓ $\tan(\theta \pm 360°) = +\tan\theta$
- Co-functions allow us to write a ratio of sin in terms of cos and vice versa.
  - ✓ $\sin(90° - \theta) = \cos\theta$
  - ✓ $\cos(90° - \theta) = \sin\theta$
  - ✓ $\sin(90° + \theta) = \cos\theta$
  - ✓ $\cos(90° + \theta) = -\sin\theta$

Example 1: Simplify without using a calculator:

$$\frac{2\sin(180° - x)\cos(360° - x)}{\cos(90° - x)\cos(180° + x)}$$

Solution:

$$\frac{2\sin(180° - x)\cos(360° - x)}{\cos(90° - x)\cos(180° + x)}$$

$$= \frac{2\sin x\cos x}{\sin x(-\cos x)}$$

$$= \frac{2}{-1}$$

$$= -2$$

Example 2: Simplify $\dfrac{\tan(180° - x)\cos(360° - x) + \cos(90° + x)}{\cos(90° - x)\sin(90° + x)}$

Solution:

$$\frac{\tan(180° - x)\cos(360° - x) + \cos(90° + x)}{\cos(90° - x)\sin(90° + x)}$$

$$= \frac{(-\tan x \cos x + (-\sin x)}{\sin x \cos x}$$

$$= \frac{\left(-\dfrac{\sin x}{\cos x}\right)(\cos x) - (\sin x)}{\sin x \cos x}$$

$$= \frac{(-\sin x) - (\sin x)}{\sin x \cos x}$$

$$= \frac{-2\sin x}{\sin x \cos x}$$

$$= \frac{-2}{\cos x}$$

Example 3: Simplify

$$\sin(-180° - \theta)\cos(-180° + \theta)\tan(-\theta) + 1 + \cos^2(\theta - 360°)$$

Solution:

$$\sin(-180° - \theta)\cos(-180° + \theta)\tan(-\theta) + 1 + \cos^2(\theta - 360°)$$

$$= \sin(-180° - \theta)\cos(-180° + \theta)\tan(-\theta) + 1 + [\cos(\theta - 360°)]^2$$

$$= (\sin\theta)(-\cos\theta)(-\tan\theta) + 1 + (\cos\theta)^2$$

$$= (\sin\theta)(-\cos\theta)\left(-\frac{\sin\theta}{\cos\theta}\right) + 1 + \cos^2\theta$$

$$= \sin^2\theta + 1 + \cos^2\theta \rightarrow \sin^2\theta + \cos^2\theta = 1$$

$$= 1 + 1$$

$$= 2$$

Example 4: Evaluate $\dfrac{\cos 120^\circ \tan 225^\circ \sin 270^\circ}{\sin(-60^\circ)\cos 480^\circ}$ without using a calculator.

Solution:

$$\frac{\cos 120^\circ \tan 225^\circ \sin 270^\circ}{\sin(-60^\circ)\cos 480^\circ}$$

$$=\frac{\cos(180^\circ-60^\circ)\tan(180^\circ+45^\circ)(-1)}{-\sin(60^\circ)\cos(360^\circ+120^\circ)}$$

$$=\frac{(-\cos 60^\circ)\tan 45^\circ.(-1)}{-\sin 60^\circ \cos 120^\circ}$$

$$=\frac{-\cos 60^\circ \tan 45^\circ.(-1)}{-\sin 60^\circ \cos(180^\circ-60^\circ)}$$

$$=\frac{\left(-\frac{1}{2}\right)(1)(-1)}{\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)}$$

$$=\frac{\sqrt{3}}{2}$$

Example 5:

Determine the value of the following in terms of p, if $\cos 63^\circ = p$, without using a calculator.

$a)$ $\sin 27^\circ$

$b)$ $\tan(-63^\circ)$

Solutions:

$a)$ $\sin 27^\circ = \cos 63^\circ$

$= p$

Exercises

1. If $\cos 23^\circ = p$; express the following in terms of p, without using a calculator.

   1.1 $\cos 203^\circ$

   1.2 $\sin 293^\circ$

2. Simplify the following expressions, without using a calculator.

   2.1 $\dfrac{\sin(90^\circ+x)\sin(-x)\tan(180^\circ+x)}{\cos(360^\circ-x)\sin(x-180^\circ)}$

   2.2 $\dfrac{\tan 300^\circ+\cos(90^\circ+x)}{\sin x+2\cos(-30^\circ)}$

2.3 $\dfrac{\cos 600° + \tan(-45°)}{3\sin 270° \times \cos 150°}$

Solutions:

1.1	$\cos 23° = p$ $\therefore x = p, r = 1$ $y = \sqrt{1 - p^2}$ $\cos 203° = \cos(180° + 23°)$ $= -\cos 23°$ $= -p$	1.2	$\sin 293° = \sin(360° - 67°)$ $= \sin 67°$ $= \sin(90° - 23°)$ $= \sin 23°$ $= \sqrt{1 - p^2}$
2.1	$\dfrac{\sin(90° + x)\sin(-x)\tan(180° + x)}{\cos(360° - x)\sin(x - 180°)}$ $= \dfrac{\cos x(-\sin x)\tan x}{\cos x(-\sin x)}$ $= \tan x$	2.2	$\dfrac{\tan 300° + \cos(90° + x)}{\sin x + 2\cos(-30°)}$ $= \dfrac{-\tan 60° - \sin x}{\sin x + 2\cos 30°}$ $= \dfrac{-\sqrt{3} - \sin x}{\sin x + 2\left(\frac{\sqrt{3}}{2}\right)}$ $= \dfrac{-\left(\sqrt{3} + \sin x\right)}{\sin x + \sqrt{3}}$ $= -1$
2.3	$\dfrac{\cos 60° + \tan(-45)}{3\sin 270° \times \cos 150°}$ $= \dfrac{\cos 270° - \tan 45°}{3(-1) \times (-\cos 30°)}$ $= \dfrac{-\frac{1}{2} - 1}{-3 \times \left(-\frac{\sqrt{3}}{2}\right)}$ $= \dfrac{-\frac{3}{2}}{\frac{3\sqrt{3}}{2}}$ $= -\dfrac{1}{\sqrt{3}}$		

Note:

sin (20° + 30°) ≠ sin 20° + sin 30°

When two angles are added or subtracted to form a new angle, then a compound angle or a double angle is formed.

A compound angle formula is an identity that expresses a trigonometric function in the form (A + B) or (A – B), in terms of trigonometric functions A and B.
Start by proving that $\cos(A+B) = \cos A \cos B - \sin A \sin B$; then use this formula to find the rest of the compound angle formulae.

Let S $(\cos A; \sin A)$ and T$(\cos B; \sin B)$ be any two points on the unit circle O.

If SÔX = A and TÔX = B then

SÔT = $A - B$

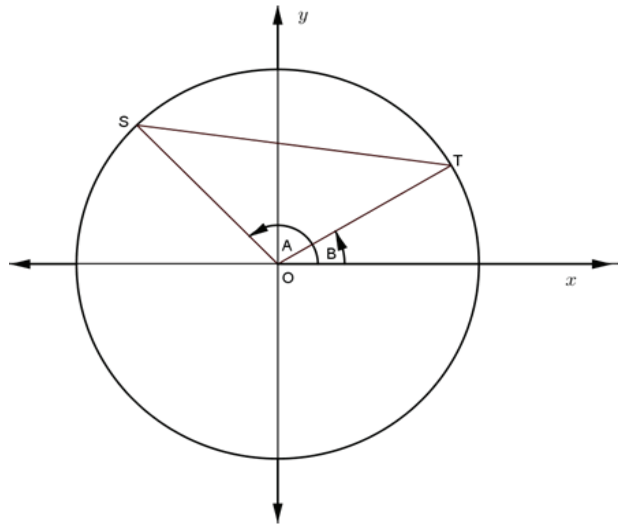
From the cosine formula:

$ST^2 = 1^2 + 1^2 - 2(1)(1)[\cos(A-B)]$

$\therefore ST^2 = 2 - 2\cos(A-B)$ .... (1)

From the distance formula:

$ST^2 = (x_S - x_T)^2 + (y_S - y_T)^2$



$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$ (**N.B**: $\sin^2 A + \cos^2 A = 1$)
$= 2 - 2\cos A \cos B - 2\sin A \sin B$ .... (2)

$\therefore\ 2 - 2\cos(A-B) = 2 - 2\cos A \cos B - 2\sin A \sin B$
$-2\cos(A-B) = -2\cos A \cos B - 2\sin A \sin B$
÷ (– 2)
$: \cos(A-B) = \cos A \cos B + \sin A \sin B$

Examples
Prove that:
cos(α + β)= cosα.cosβ – sinα.sinβ

Solution:
cos(α + β) = cos[α – (–β)] = cosα.cos(–β) + sinα.sin(–β)
= cosα.cosβ + sinα.(–sinβ)
= cosα.cosβ – sinα.sinβ

Prove that:
sin(α + β)= sinα.cosβ + cosα.sinβ
Solution:
sin(α + β) = cos[90° – (α + β)] = cos[90° – α – β] = cos[(90° – α) – β]
= cos(90° – α). cos(β) + sin(90° – α). sin(β)
= sinα.cosβ + cosα.sinβ
Prove that:
sin(α – β= sinα.cosβ – cosα.sinβ

Solution:
sin(α – β) = cos[90° – (α – β)] = cos[90° – α + β] = cos[(90° + β) – α]
= cos(90° + β). cosα + sin(90° + β). sinα
= –sinβ.cosα + cosβ.sinα
= sinα.cosβ – cosα.sinβ
Simplify, without the use of a calculator:
.1 cos70° cos10° + cos20° cos80°
.2 2 sin15° cos 15°
.3 sin 15°

Solutions:
.1 $cos\ 70^0 cos10^0 + sin20^0 cos80^0$
$= cos\ 70^0 cos10^0 + sin70^0 cos10^0$
$= cos\ 70^0 - 10^0$
$= cos\ 60^0$
$= \frac{1}{2}$
.2 $2 = sin\ 15^0 cos15^0$
$= sin2(15^0)$
$= sin30^0$
$= \frac{1}{2}$
.3 $sin\ 15^0$
$= sin(45^0 - 30^0)$
$= sin\ 45^0 cos30^0 + cos45^0 sin30^0$
$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$
Prove the following:
$\sin 2A = 2\sin A.\cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = \cos^2 A - \sin^2 A$

Prove the identity $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = \frac{4\tan x}{\cos x}$

Hints:

- Simplify the LHS.
- Get an LCD.
- Then simplify.

Without using the calculator, simplify:

$2\cos 5\theta \cos 4\theta - \sin 9\theta$

$\cos 350° \sin 40° - \cos 440° \cos 40°$

$\frac{\sin 63°}{\sin 21°} - \frac{\cos 63°}{\cos 21°}$

Answers

$\sin \theta$

$\frac{1}{2}$

2

Evaluate, without using a calculator:

$cos^2 15° + \sin 22{,}5° \cos 22{,}5° - sin^2 15°$

$(1 - \sqrt{2}\cos 15°)(1 - \sqrt{2}\cos 195°)$

Answers

$\frac{2\sqrt{3}+\sqrt{2}}{4}$

$\frac{\sqrt{3}}{2}$

General solutions to trigonometric equations

Concepts and skills

- Simplify trigonometric equations.
- Find the reference angle.
- Use reduction formulae to find other angles within each quadrant.
- Find the general solution of a given trigonometric equation.

Prior knowledge:

- Trigonometric ratios.
- Trigonometric identities.
- Solve problems in the Cartesian plane.
- Co-functions.
- Factorisation.
- Revise the following trigonometric ratios for right-angled triangles, which you learnt in Grade 10.

  ✓ $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

✓ $\cos\theta = \dfrac{\text{Adjecent}}{\text{Hypotenuse}}$

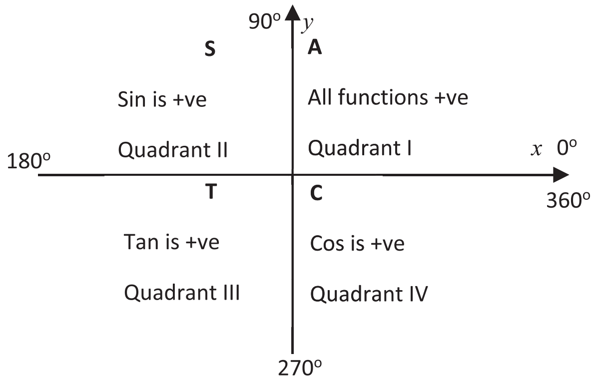
✓ $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

- **Factorising trigonometric expressions**
  - ✓ Trigonometric expressions are factorised in the same way as factorising algebraic expressions such as: differences of two squares: $x^2 - y^2 = (x-y)(x+y)$

    some trinomials: $x^2 + 3x + 2 = (x+1)(x+2)$

- **Trigonometric identities**
  - ✓ $\sin^2\theta + \cos^2\theta = 1$
  - ✓ $\sin^2\theta = 1 - \cos^2\theta$
  - ✓ $\cos^2\theta = 1 - \sin^2\theta$
  - ✓ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
  - ✓ $\dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

- **Trigonometric ratios in a Cartesian plane**
  - ✓ The CAST diagram shows the ratios that are positive in each quadrant and gives a summary of the signs of each ratio.



- **Co-functions allow us to write a ratio of sin in terms of cos and vice versa**
  - ✓ $\sin(90^\circ - \theta) = \cos\theta$
  - ✓ $\cos(90^\circ - \theta) = \sin\theta$
  - ✓ $\sin(90^\circ + \theta) = \cos\theta$
  - ✓ $\cos(90^\circ + \theta) = -\sin\theta$

Trigonometric equations - hints:

- A trigonometric equation is true only for certain values of θ.
- Write the equation on its own on one side of the equation.
- Start by simplifying an equation as far as possible. Use identities, double and compound angle formulae, and factorisation, where possible. You need *one* trig ratio and *one* angle equal to a *constan*t, for example $\cos x = \frac{1}{2}$.
- Find the reference angle.
- Identify the possible quadrants in which the terminal rays of the angles could be, based on the sign of the function.
- Consider restrictions or intervals for specific solutions.

  For general solutions:
- Because trig functions are periodic, there will be a number of possible solutions to an equation. You will need to write down the **general solution** of the equation.
- Once the solution has been solved, write down the general solution by adding the following:

  $+ k360^0$ for cosine and sine, because they repeat every 360^0.

- If you add or subtract any number of full revolutions to a solution, you get the same number, i.e.:

  If $\sin x = a$, $-1 \le a \le 1$. Then $x = \sin^{-1} a + k360° \; or \; x = 180° - \sin^{-1} a + k360°, k \in Z$

  If $\cos x = a$, $-1 \le a \le 1$. Then $x = \pm\cos^{-1} a + k360°,\; k \in Z$

  $+ k180^0$ for tangent, because it repeats every 180^0, i.e.

  If $\tan x = a, a \in \Re$. Then $x = \tan^{-1} a + k180°, k \in Z$
- If an equation contains double angles, for example, $\sin 3\theta$, $\cos 2x$ and $\tan 5y$, first find the general solutions for 3θ, $2x$ and $5y$. Then divide by 3, 2 or 5 to find the final solutions. If you divide first, you will lose valid solutions.

- Apply restrictions.

Example 1: Determine the general solution for: $\cos\theta = -0{,}766°$

Solution:

$$\cos\theta = -0{,}766°$$

$$\theta = \cos^{-1}(-0{,}766)$$

$$= \pm 139{,}99° + k360°, k \in Z$$

Example 2: Solve for *x*: $\cos 2x = -0{,}174°$ for $-180° \le x \le 180°$

Solution:

$$2x = \cos^{-1}(-0{,}174)$$
$$\text{Ref}\angle = 100{,}02° \cong 100°$$

$\therefore 2x = 100° + n360°$ **and** $2x = -100° + n360°$

$x = 50° + n180°$	$x = -50° + n180°$

Solve these equations for $n = \ldots -2; -1; 0; 1; 2 \ldots$ and select values that lie in $[-360°; 360°]$

Answer: $x = -310°; -230°; 130°; 50°; 130°; 230°$

Example 3: Solve for *x*: $\sin(3x + 50°) + \cos(2x - 10°) = 0$. Hence, determine *x* if $x \in [-180°; 180°]$

Solution:

$$\sin(3x + 50°) = -\cos(2x - 10°)$$
$$\sin(3x + 50°) = -\sin[90° - (2x - 10°)]$$
$$\sin(3x + 50°) = -\sin(100° - 2x)$$
$$\sin(3x + 50°) = \sin(2x - 100°)$$
$$\therefore 3x + 50° = 2x - 100° \ \ or \ \ 3x + 50° = 180° - (2x - 100°) + k360°$$
$$\therefore x = 150° + k360°$$

$\therefore 3x + 50° = 2x - 100°$ $\therefore x = 150° + k360°$	Or	$3x + 50° = 180° - (2x - 100°) + k360°$ $3x + 50° = 180° - 2x + 100° + k360°$ $5x = 230° + k360° \ \ k \in Z$ $x = 46° + k72° \ \ k \in Z$

$$\therefore x = 46°; 118°; -26°; -98°; -170°; -150°$$

Example 2: Solve for *x*: $\cos 2x = -0{,}174°$ for $-180° \leq x \leq 180°$

Solution:

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Solution:

$\sin(3x + 50°) = -\cos(2x - 10°)$
$\sin(3x + 50°) = -\sin[90° - (2x - 10°)]$
$\sin(3x + 50°) = -\sin(100° - 2x)$
$\sin(3x + 50°) = \sin(2x - 100°)$
$\therefore 3x + 50° = 2x - 100° \;\; or \;\; 3x + 50° = 180° - (2x - 100°) + k360°$
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$\therefore x = 46°; 118°; -26°; -98°; -170°; -150°$

Example 4: If $\theta \in [0°; 180°]$, solve for θ, correct to one decimal place: $3\sin 2\theta = -2{,}34$

Solution:

$3\cos 2\theta = -2{,}34$

$\cos 2\theta = -0{,}78$

$2\theta = \pm\cos^{-1}(-0{,}78) + k360°$

$2\theta = \pm 141{,}26° + k360°$

$\theta = \pm 70{,}6° + k360°$

Solution:

$2\tan x - 3 = \cos 32°$

$2\tan x - 3 = 0{,}84...$

$\tan x = 1{,}92...$

$x = \tan^{-1}(1{,}92...)$

$= 62{,}54°$

Example 6:

Determine the general solution of:

$2\sin B\cos B - 4\cos B = \sin B - 2$

$2\sin B\ \cos B - 4\cos B - \sin B + 2 = 0$

$2\cos B(\sin B - 2) - (\cos B - 2) = 0$

$\therefore (\sin B - 2)(2\cos B - 1) = 0$

$\therefore \sin B = 2$ or $\cos B = \frac{1}{2}$

no solutions or $\hat{B} = \mathbf{60° + k360°}, k \in \mathrm{Z}$

$\hat{B} = (360° - 60°) + k\ 360° = \mathbf{300° + k360°}\ k \in \mathrm{Z}$

Practice exercises

1. Determine the value of *x*:

1.1 $\sin 2x = \frac{1}{2}$ for $x \in [0°;90°]$

1.2 $4\cos^2 x - \tan 45° = 0$ for $x \in [0°;360°]$

2. Find the general solution of $\sin(\theta + 14{,}5°) = 0{,}609$ correct to one decimal place.

3. Use factorisation to find the general solution of $2\cos^2\theta + \cos\theta = 0$.

4. Determine the value of:

$\sqrt{\tan\theta} = x + \frac{1}{x}$ if $x^2 + \frac{1}{x^2} = 1$

5. Determine the solution/s of the equation.

Solutions for the practice exercises

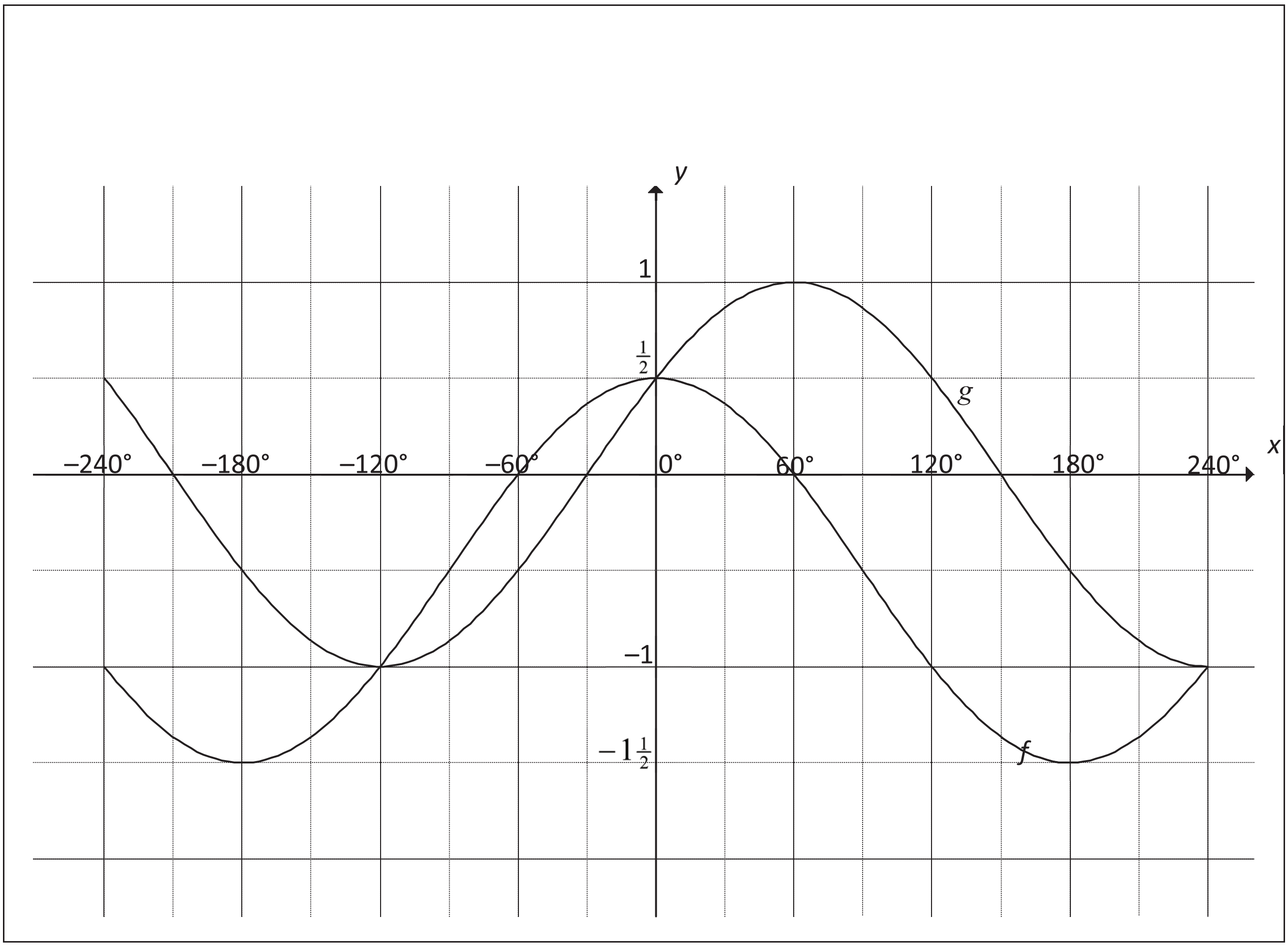
1.1	$\sin 2x = \frac{1}{2}$ Let $A = 2x$ $\sin A = \frac{1}{2}$ $\hat{A} = \sin^{-1}\left(\frac{1}{2}\right)$ $= 30°$ $2x = 30°$ $x = 15°$	**1.2**	$\tan 45° = 1$ $\therefore 4\cos^2 x - 1 = 0$ $\cos^2 x = \frac{1}{4}$ $\cos x = \pm\sqrt{\frac{1}{4}}$ $\therefore \cos x = \pm\frac{1}{2}$ Ref $\angle = 60°$ $x = \pm 60° + k\,360°\;\; k \in Z$ $\therefore x = 60°; 120°; 240°\text{or } 300°$
2	$\sin(\theta + 14{,}5°) = 0{,}609$ $\theta + 14{,}5° = 37{,}5° + n\,360°, n \in Z$ or $\theta + 14{,}5° = 142{,}5° + n\,360°, n \in Z$ $\therefore \theta = 23° + n\,360°$ or $\theta = 128° + n\,360°, n \in Z$	**3**	$2\cos^2\theta + \cos\theta = 0$ $\cos\theta(2\cos\theta - 1) = 0$ $\cos\theta = 0$ or $\cos\theta = \frac{1}{2}$ $\therefore \theta = 90° + n180°$ or $\theta = \pm 60° + n360°, n \in Z$
4	$\left(\sqrt{\tan\theta}\right)^2 = \left(x + \frac{1}{x}\right)^2$ $\tan\theta = x^2 + 2 + \frac{1}{x^2}$ $= x^2 + \frac{1}{x^2} + 2$ $= 1 + 2$ $= 3$ Ref $\angle = 71{,}56°$ $\theta = 71{,}56° + k180°\;\; k \in Z$		

Trigonometric graphs

Know how to sketch and interpret the graphs of sine, cosine and tangent.

Example

1. In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \leq x \leq 240°$. The graphs intersect at $\left(0° ; \frac{1}{2}\right)$, (–120°; –1) and (240°; –1).



1.1 Determine the values of p and q.

1.2 Determine the values of x in the interval $-240° \leq x \leq 240°$, for which $f(x) > g(x)$.

1.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$.

Solutions

1.1 $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30°)$

$\therefore\ p = 30°$ and $q = -\frac{1}{2}$

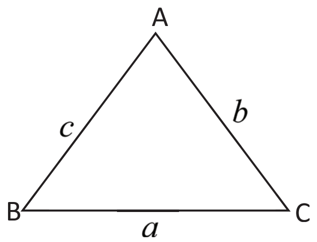
1.2 $x \in (-120°\ ;\ 0°)$ **OR** $-120° < x < 0°$

1.3 The graph of *g* has to shift 60° to the left and then be reflected on the *x*-axis.

Solving sides and angles in triangles that are not right-angled triangles: sine, cosine and area rule

Rule	**Formula**	**When to use**
Sine rule	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Given two sides and the angle opposite one of those sides. One side and any two angles.
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$	Given two sides and the included angle. Three sides.
Area rule	$\frac{1}{2}ab\sin C$	Area is required. In order to use the formula for Area, two sides and the included angle are required.

Naming angles and sides in a triangle



Sine rule

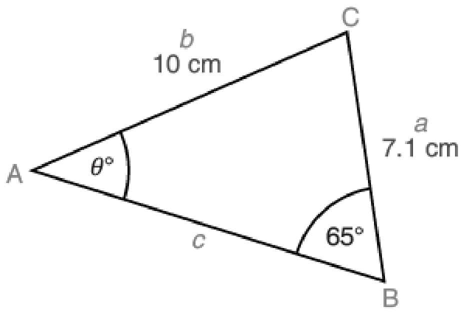
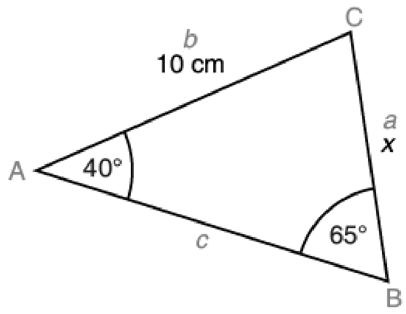
1. You should use the sine rule when:

Case 1: you know *two sides and an angle* and you want to find the size of an **angle** opposite a known side.

Case 2: you know *two angles and one side and want* to get the **side** opposite a known angle.

2. In both cases, you must already know a side and an angle that are opposite each other (one full pair).

3. Note the importance of the word *opposite*.

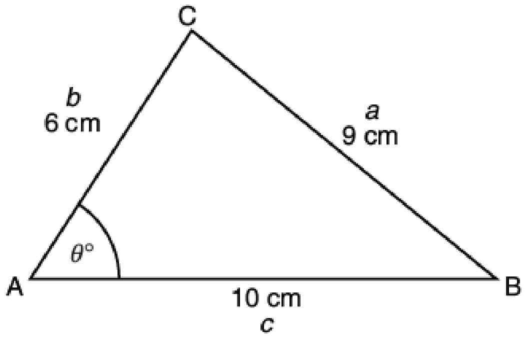
Case 1:	**Case 2:**
 	
Solution: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{\sin A}{7.1} = \frac{\sin 65^{\circ}}{10}$ (Note how one full pair is known and one pair in the formula is unnecessary) $\sin A = \frac{7.1 \sin 65^{\circ}}{10}$ $\therefore \hat{A} = 40.05^{\circ}$ (Calculator work: shift → sin → $\frac{7.1 \sin 65^{\circ}}{10}$)	Solution: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Note that as we are looking for a side we can use this version of the sine rule. It is easier to have the unknown in the numerator position) $\frac{x}{\sin 40^{\circ}} = \frac{10}{\sin 65^{\circ}}$ $x = \frac{10 \sin 40^{\circ}}{\sin 65^{\circ}}$ $\therefore x = 7{,}09$

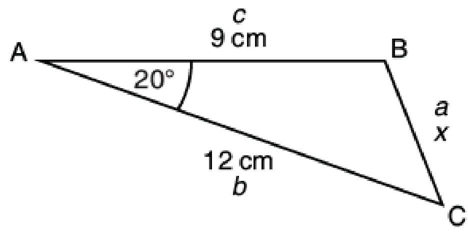
Cosine rule

1. You should use the cosine rule when:

Case 1: you know *three sides* and you want to find the size of an **angle**

Do the following worked examples on the board with learners:

<table>
<tr><th>Case 1</th><th>Case 2</th></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>
Solution:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$9^2 = 6^2 + 10^2 - 2(6)(10)\cos A$$
$$81 = 36 + 100 - 120\cos A$$
$$-55 = -120\cos A$$
$$\frac{55}{120} = \cos A$$
$$\therefore \hat{A} = 62{,}72^\circ$$
</td><td>
Solution:

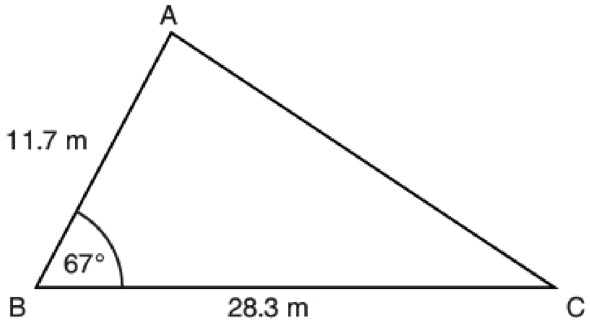
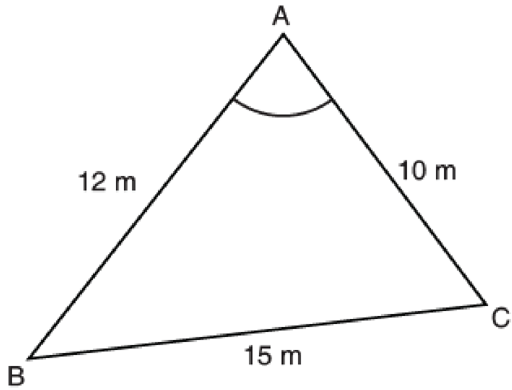
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 12^2 + 9^2 - 2(12)(9)\cos 20^\circ$$
$$x^2 = 22{,}026\ldots$$
$$x = 4{,}693 \text{ cm}$$
</td></tr>
</table>

Case 2: you know *two sides and the included angle and want* to find a **side or an angle**.

Area rule

1. You should use the area rule when you have been asked to find the area of a triangle. You will need two sides and the included angle.
2. If the correct information is not available, use the sine rule or cosine rule first.

Do the following worked examples on the board with learners:

Case 1 (two sides and the included angle available)	Case 2 (two sides and included angle not available)
	
$\text{Area} = \frac{1}{2}ac\sin C$ $\text{Area} = \frac{1}{2}(28{,}3\text{ m})(11{,}7\text{ m})\sin 67°$ $\therefore \text{Area} = 152{,}39\text{ m}^2$	To find $\hat{A}$: $15^2 = 10^2 + 12^2 - 2(10)(12)\cos A$ $225 = 100 + 144 - 240\cos A$ $-19 =- 240\cos A$ $\cos A = \frac{19}{240}$ $\therefore \hat{A} = 85{,}46°$ $\text{Area} = \frac{1}{2}bc\sin A$ $\text{Area} = \frac{1}{2}(10\text{ m})(12\text{ m})\sin 85{,}46°$ $\therefore \text{Area} = 59{,}81\text{ m}^2$

2D and 3D problems

Trigonometry was developed in ancient civilisations to solve practical problems such as those found in building construction and when navigating by the stars. We will show that trigonometry can also be used to solve some other practical problems. We use the trigonometric functions to solve problems in two

dimensions that involve right-angled triangles.

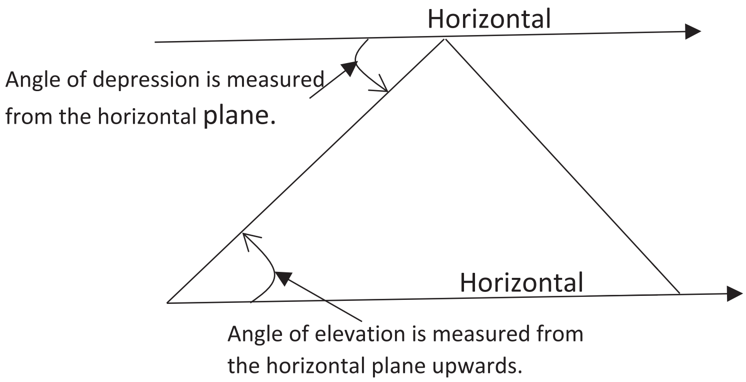
Concepts and skills

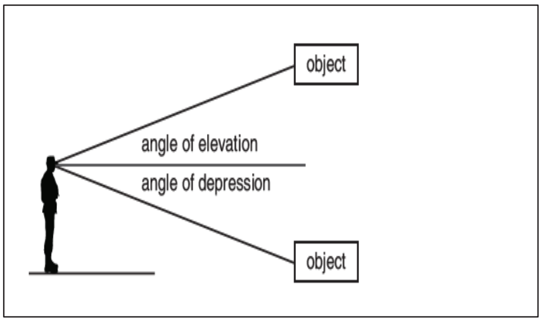
- ✓ Solve problems in two and three dimensions.
- ✓ Identify the angle of elevation.
- ✓ Identify the angle of depression.

Prior knowledge:

- Trigonometric ratios
- Pythagoras theorem
- Sine rule
- Cosine rule
- Area formula
- Solving 2-dimensional triangles
- Compound angles
- Reduction formulae

Angles of elevation and depression are measured in the vertical plane from the horizontal plane.



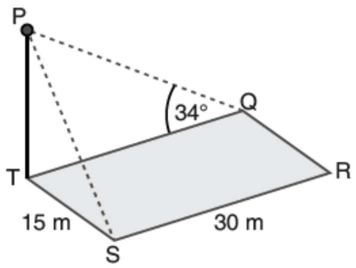


In the following diagram, a vertical flagpole (TP) stands in the corner of a field (QRST).

Using the information given, calculate:

a. the height of the flagpole

b. the angle of elevation of P from S.



therefore, I will use tan

a) In ΔPQT,
QT = 30m (opposite sides of a rectangle)

$$\tan 34^\circ = \frac{PT}{30}$$
$$30 \tan 34^\circ = PT$$
$$\therefore PT = 20,2 \text{ m}$$

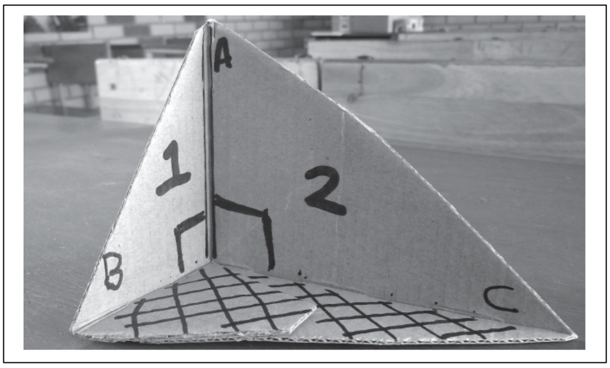
In PTS

$$\tan \hat{S} = \frac{20,2}{15}$$
$$\therefore \hat{S} = 53,4^\circ$$

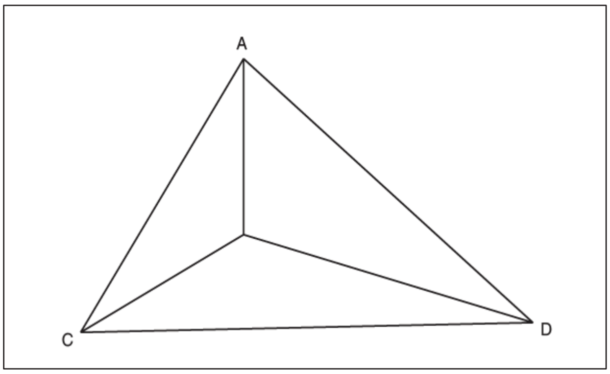
12. In conclusion, there are **four** triangles in the above diagram that could be considered in order to solve the problem:
   - The two right-angled triangles on each side.
   - The triangle on the ‘floor/ground’.
   - The imaginary triangle formed ‘in the air’ in front of the represented situation.

A corner of a box can be used to visualise the planes in a 3-dimensional object

1. Cut the corner off a sturdy box and cut pieces off the sides in order to have two triangles on each side of the corner.
2. Shade the triangle at the bottom a different colour.



OR

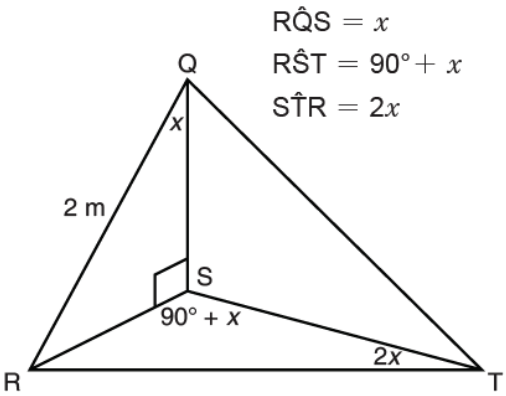


Tips for solving 3-D problems

1. These tips will assist you to solve a problem. You should write these down in your exercise books.
2. The following steps will help you to solve a 3-dimensional problem:
   - Fill in ALL possible information on the diagram (even if it means naming an angle $(90° - x)$ or $(x + y)$). A thorough knowledge of Grade 8 geometry theorems is essential.
   - Shade the triangle that represents the horizontal plane (in other words, the part of the diagram that represents the floor/ ground). This will help you to visualise the 3-dimensional form.

     Look for ‘separate’ triangles to see which ones present enough information to use the sine or cosine rule in order to find more sides or angles.
   - Remember to use basic trigonometry from Grade 10 in right-angled triangles.
   - Make use of the sine rule wherever possible. It is the simplest rule to use.

Examples

<table>
<tr>
<td>
Two boys (Phila and Thabani) are standing at points R and T. Each boy is looking up at point Q, which is the top of a vertical pole, SQ. Phila (R) is standing 2 metres from the bottom of the pole. S, R and T are in the same horizontal plane.


a) Prove that Phila and Thabani are standing one metre apart.
b) Find SR̂ T in terms of *x*.
c) Prove that ST = 2 cos 2*x* - 1.
</td>
<td>
Note that BOTH angles at S (RŜQ and TŜQ) are right angles, as the pole is straight.
Shade in the horizontal plane (DRST).
To answer (a), we need to show that RT = 1.
In order to do that, we need to first work in DRSQ and find what RS is equal to, so that we can move into DRST and use this information (obtained earlier on).

Note that, once again, we are looking for shared sides and the opportunity to work in a triangle that has sufficient information and not too many unknowns.
(b) requires knowledge of Grade 8 geometry. In this case, angles of a triangle = 180°.
Note that when it is a relatively simple question, you usually need to use angles and sides to answer the question.
(c) Firstly, when asked to prove something, you need to approach the question as if you have been asked to find ST.
Once you have found ST, you can look at the question and check if it is correct. ST is in a triangle where there are now four pieces of information (3 angles and a side).
You should be asking yourself if you can use the sin rule or the cos rule to find ST.
As the angle opposite ST is known (it was just found in (b)), and there is another full pair of ‘opposites’ (RT and RŜT, the sin rule will work.

</td>
</tr>
</table>

Solution:

a) In ΔRQS:

$$\sin x = \frac{\text{RS}}{2}$$
$$\therefore \text{RS} = 2\sin x$$

In ΔRST:

$$\frac{\text{RT}}{\sin(90^\circ + x)} = \frac{\text{RS}}{\sin 2x}$$
$$\frac{\text{RT}}{\sin(90^\circ + x)} = \frac{2\sin x}{2\sin x \cos x}$$
$$\frac{\text{RT}}{\cos x} = \frac{1}{\cos x}$$
$$\therefore \text{RT} = 1$$

b) $\text{S}\hat{\text{R}}\text{T} = 180^\circ - (90^\circ - x) - 2x$ (<'s of a Δ)

$\therefore \text{S}\hat{\text{R}}\text{T} = 90^\circ - 3x$

c) In ΔRST:

$$\frac{\text{ST}}{\sin(90^\circ - 3x)} = \frac{\text{RT}}{\sin(90^\circ + x)}$$
$$\frac{\text{ST}}{\sin(90^\circ - 3x)} = \frac{1}{\sin(90^\circ + x)}$$
$$\text{ST} = \frac{1.\,sin(90^\circ - 3x)}{\sin(90^\circ + x)}$$
$$\text{ST} = \frac{\cos 3x}{\cos x}$$
$$\text{ST} = \frac{\cos(2x + x)}{\cos x}$$
$$\text{ST} = \frac{\cos 2x\ \cos x - \sin 2x\ \sin x}{\cos x}$$
$$\text{ST} = \frac{\cos 2x\ \cos x - 2\sin x\ \cos x\ \sin x}{\cos x}$$
$$\text{ST} = \frac{\cos \text{x}(\cos 2\text{x} - 2\sin^2 x)}{\cos x}$$
$$\text{ST} = \cos 2\text{x} - 2\sin^2 x$$
$$\text{ST} = \cos 2x - (1 - \cos 2x)$$
$$\text{ST} = \cos 2x - 1 + \cos 2x$$
$$\text{ST} = 2\cos 2x - 1$$

5 Message to Grade 12 learners from the writers

Mathematics can be fun, as it requires you to pull together all that you have learnt in the lower grades to answer the Grade 12 examination questions. If you skipped one grade before Grade 12, it would have left a void in the grounding you require to pass the final examinations.

Please ensure that you know all axioms and corollaries (all the rules) to answer the questions.

Revise by working through at least three sets of previous Mathematics question papers before you sit the final examinations.

Write one paper in 3 hours and mark your script on your own, using the memorandum, in order to gauge whether you are ready for the final paper. Memoranda for all previous DBE examinations are available on the DBE website.

We assure you that this year's final paper will be similar to those of previous years in both format and style.

Limit the amount of food you eat, in order for your cerebrum (brain) to work effectively. Digestion occupies the functioning of your brain.

Blast the final paper.

6 Thank you and acknowledgements

We hope the guidance we have provided helps you in the final examination.

Mr Leonard Gumani Mudau, Mrs Nonhlanhla Rachel Mthembu, Ms Thandi Mgwenya and Mr Percy Steven Tebeila all wish you well.

MATHEMATICS
TRIGONOMETRY
GRADE 12

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