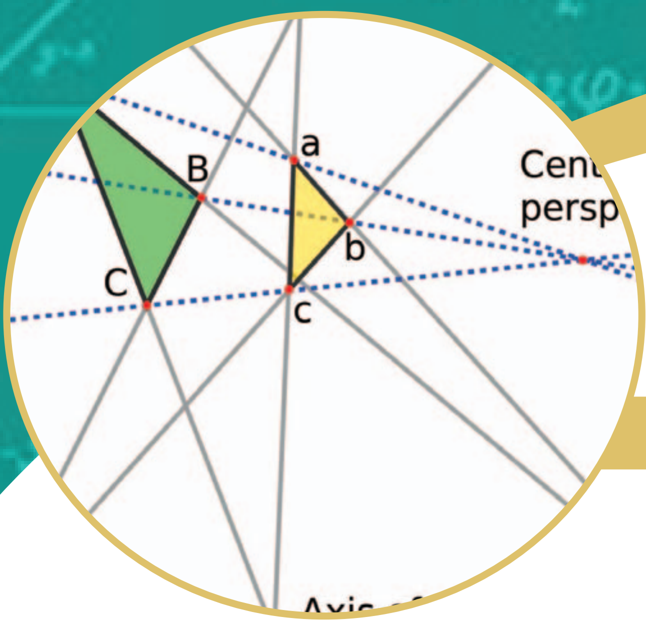


MATHEMATICS

FUNCTIONS

GRADE 12

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Read to Lead
A Reading Nation is a Leading Nation

2030 NDP

Table of Contents

2. How to use this booklet

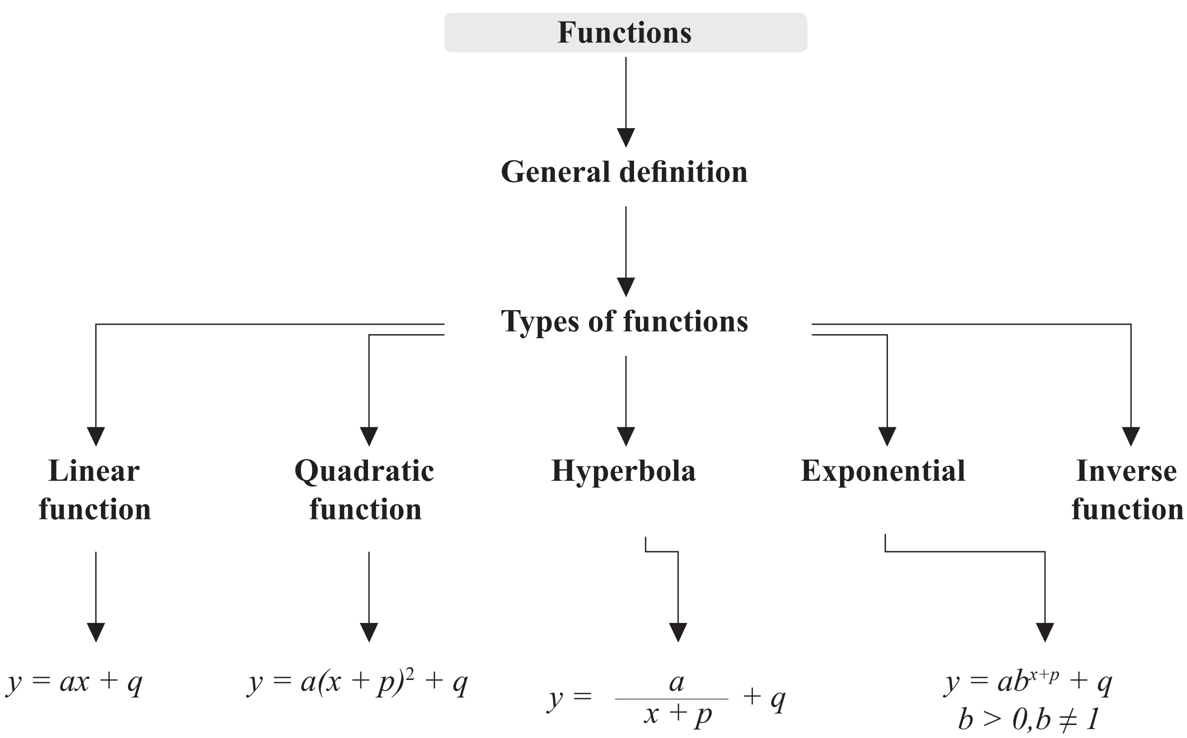
This booklet is designed to clarify the content prescribed for Mathematics. In addition, it also provides some tips on how you should tackle real problems on a daily basis. Candidates will be expected to have mastered the content outlined for grades 8-11. This booklet must be used to master some mathematical rules you may not aware of. The prescribed textbook must also be used.

3. Study and examination tips

All learners should acquire sufficient understanding and knowledge to do the following:

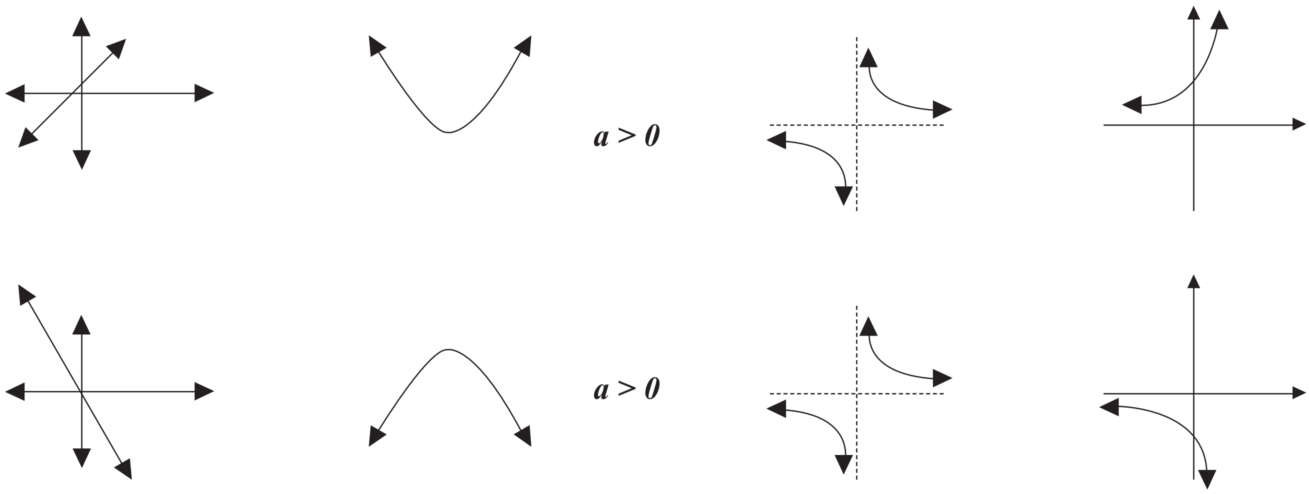
- Develop fluency in computation skills without relying on the use of a calculator.
- Generalize, make conjectures and try to justify or prove them.
- Develop problem-solving and cognitive skills.
- Make use of the language of Mathematics.
- Identify, investigate and solve problems creatively and critically.
- Use the properties of shapes and objects to identify, investigate and solve problems creatively and critically.
- Encourage appropriate communication by using descriptions in words, graphs, symbols, tables and diagrams.
- Practice Mathematics every day.

4. OVERVIEW OF FUNCTIONS



Properties of functions:
Axis of symmetry
Domain
Range
Notation

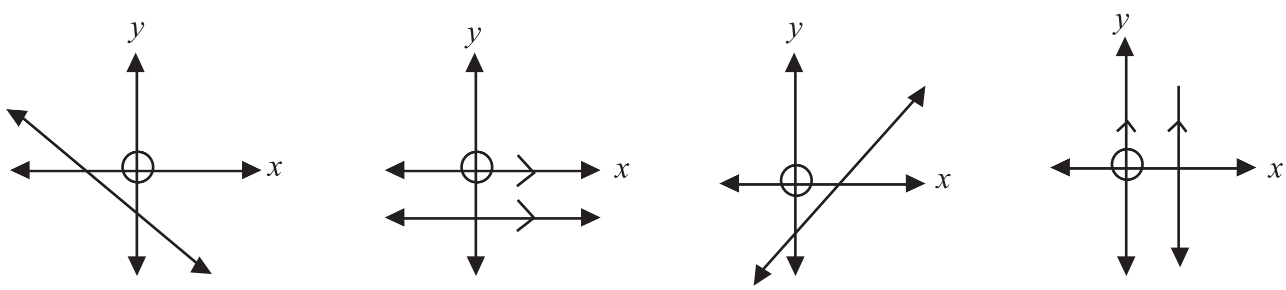
$y = ax + q$	$y = a(x + p)^2 + q$	$y = \frac{a}{x+p} + q$	$y = ab^{x+p} + q$ $b > 0, b \neq 1$



5.1 STRAIGHT LINE

General representation or equation
$y = ax + q$ or $y = mx + x.$

a or m is the gradient and q or c is the y - intercept
Also note the shape of the following linear functions:



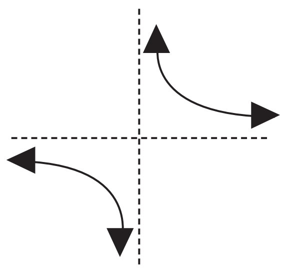
$a < 0$ $q < 0$	$a = 0$ $y = q$	$a > 0$ $q < 0$	a is undefined there is no q-value

Domain and range is $x \in \mathbb{R}$ and $y \in \mathbb{R}$ respectively.

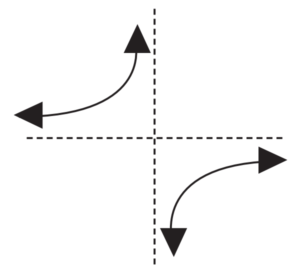
5.2 HYPERBOLA

General representation or equation

$y = \dfrac{a}{x}$ **or** $y = \dfrac{a}{x} + q$ **or** $y = \dfrac{a}{x - p} + q$ **or** $y = \dfrac{a}{x + p} + q$



Dotted lines are asymptotes



Dotted lines are asymptotes

- q is the vertical translation
- p is the horizontal translation

- For y = $\frac{a}{x}$, $p = 0$ and $q = 0$. The vertical asymptote is   and the horizontal asymptote is $y = 0$. The axes of symmetry are $y = x$ (Positive) and $y = -x$ (Negative).

  Domain is $x \neq 0, x \in \mathbb{R}$

  Range is $y = \neq 0, y \in \mathbb{R}$

- For $y = \frac{a}{x} + q$, $p = 0$. The vertical asymptote is x = 0 and the horizontal asymptote is $y = q$. The axes of symmetry are $y = x + q$ (Positive) and y = x + q (Negative).

  Domain is $x \neq 0, x \in \mathbb{R}$

  Range is $y = \neq 0, y \in \mathbb{R}$

- For $y = \frac{a}{x} + q$, $p = 0$ the vertical asymptote is $x = 0$ and the horizontal asymptote is $y = q$. The axis of symmetry is .

  Domain is $x \neq - p, x \in \mathbb{R}$

  Range is $y \neq q, y \in \mathbb{R}$

- For $y = \frac{a}{x} + p + q$, $> (y - q)(x + p) = a$ the vertical asymptote is $x + - p$ and the horizontal asymptote is $y = q$. The axis of symmetry is $y = \pm (x + p) + q$.
  - Domain is $x \neq - p, \ x \in \mathbb{R}$
  - Range is $y \neq q, y \in \mathbb{R}$

Example 1

Given $f(x) = \dfrac{3}{x - 2} + 1$

1.1 Write down the equations of the asymptotes of f.

1.2 Determine: the coordinates of B; the x-intercept of f.

1.3 Determine: the coordinates of D; the y-intercept of f.

1.4 Determine the domain and the range of f.

1.5 Determine the decreasing and increasing functions of the axes of symmetry of f.

1.6 Draw a sketch graph of f.

Solution:

1.1 $y = 1$

$x = 2$

1.2

$$y = \frac{3}{x - 2} + 1$$

$$0 = \frac{3}{x - 2} + 1$$

$$-x + 2 = 3$$

$$x = -1$$

1.3

$$y = \frac{3}{0 - 2} + 1$$

$$y = -\frac{1}{2}$$

1.4 Domain: $x \in \mathbb{R}, x \neq 2$

Range: $y \in \mathbb{R}\ y \neq 1$

1.5 Increasing axis of symmetry: $y = x + c$

$1 = 2 + c$

$c = -1$

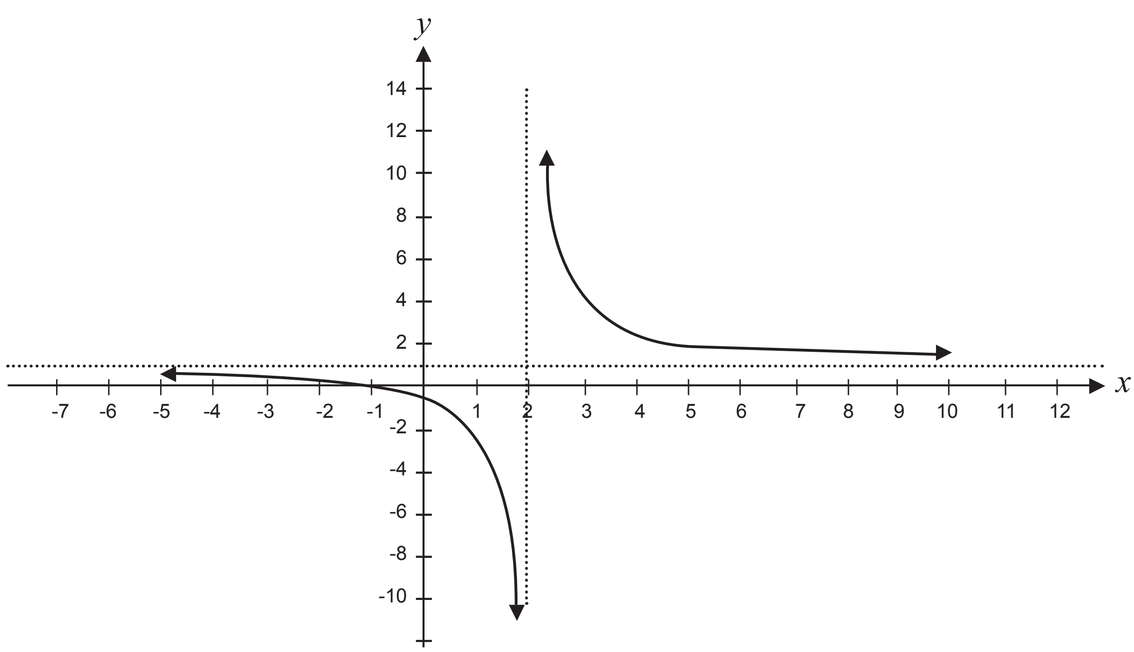
$y = x - 1$

Decreasing axis of symmetry: $y = x + c$

$1 = -2 + c$

$y = -x + 3$

1.6



Activity 2.1

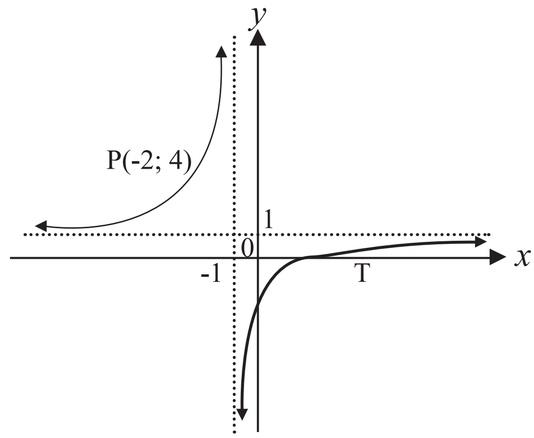
1. Given: $h(x) = \dfrac{12}{x - 4} + 6 \ \ for \ \ x > 0$

1.1 Draw a neat sketch graph of h in your workbook. Show all intercepts with the axes and asymptotes.

1.2 Write down the equation of *k* if *k* is a reflection of h about the *x*-axis.

2. The diagram above shows the graph for $f(x) = \dfrac{a}{x+p} + q$.

The lines $x = -1$ and $y = 1$ are the asymptotes of *f*. P (-2; 4) is a point on f and T is the *x*–intercept of *f*.



2.1 Determine the values of a, p, and q.
2.2 Calculate: the coordinates of T; the *x*–intercept of *f*.
2.3 Determine the value of c if the graph of f is symmetrical with respect to the line .

5.3 PARABOLA

Note:

- You need to know how to solve quadratic equations, in order to be able to deal with parabola.
- The standard form of a quadratic equation is $ax^2+bx + c = 0$
- If a quadratic equation is already factorized and one side is equal to zero, then the equation is in standard form and you should just write the answers from the given factors.
- If one side is factorized and the other side is not equal to zero, then first write the question in standard form and then factorize.
- When the question says correct to one or two decimal places, you are expected to solve the quadratic equation using the quadratic formula.
- If you struggle to find factors of a quadratic equation by inspection, use the quadratic formula
- For the inequality, simplify so that the right-hand side is 0.
- Then use the graphic (draw the sketch of the parabola) or number line method.

Examples:

1 Solve for *x:*

$x(x-1) = 0$

Solution:

$x = 0$ or $x-1= 0$

$x = 0$ or $x= 1$

2 Solve for x:

$(x-3)(x+5) = 9$

$x^2 + 5x - 3x - 9 = 0$

$x^2 + 2x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4^2 - 4(1)(-9)}}{2(1)}$$

$x = 2{,}16$ or $x = -5{,}16$

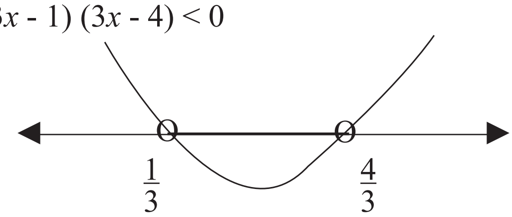
3 Solve for x:

$15x - 4 > 9x^2$

$15x - 4 - 9x^2 > 0$

$9x^2 - 15x + 4 < 0$

$(3x - 1)(3x - 4) < 0$



$$\frac{1}{3} < x < \frac{4}{3}$$

Activity 3.1

1. Given $x^2 + 2x = 0$

   1.1 Solve for x
   1.2 So, you must determine the values of x for which $x^2 \geq -2x$

2. Solve for x:
   $2x^2 - 3x - 7 = 0$ (correct to TWO decimal places)

3. Solve for x
   $\sqrt{x - 2} + x = 4$

ANSWERS

1.1 $x = 0$ or $x = -2$

1.2 $x \leq -2$ or $x \geq 0$

2. $x = 2{,}77$ or $x = -1{,}27$

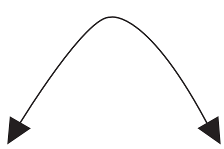
3. $x = 3$ is the only solution

3.2 General representation or equation

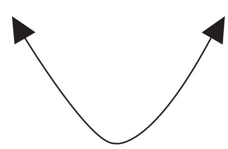
$y = ax^2$ *or* $y = ax^2 + q$ *or* $y = a\,(x + p)^2 + q$ *or* $y = ax^2 + bx + c$

Important deductions

for $a < 0$



for $a > 0$



- For $y = ax^2$, $p = 0$ and $q = 0$, the **turning point** is (0;0) and **y-intercept** is $y = 0$
  The domain is $x \in \mathbb{R}$ and **the range** is $y \geq 0; y \in \mathbb{R}$ if $a > 0$ or $y \leq 0; y \in \mathbb{R}$ or R if $a < 0$
- For $y = ax^2 + q$, $p = 0$, the **turning point** is (0;q) and **y-intercept** is $y = q$
  The domain is $x \in \mathbb{R}$ and **the range** is $y \geq q; y \in \mathbb{R}$ if $a > 0$ or $y \leq 0; y \in \mathbb{R}$ if $a < 0$
- For $y = a\,(x + p)^2 + q$, the **turning point** is (-p; q) and **y-intercept** is $y = a\,(p)^2 + q$
  The domain is $x \in \mathbb{R}$ and **the range** is $y \geq q; y \in \mathbb{R}$ if $a > 0$ or $y \leq 0; y \in \mathbb{R}$ if $a < 0$

- For $y = ax^2 + bx + c$, the **turning point** is $\left(\frac{-b}{2a}; \frac{4ac - b^2}{4a}\right)$ and **y-intercept** is

  $y = c$

- The **domain** is $x \in \mathbb{R}$ and the **range** is $y \geq \frac{4ac - b^2}{4a}$; $y \in \mathbb{R}$ *if* $a > 0$ or

  $y \leq \frac{4ac - b^2}{4a}$; $y \in \mathbb{R}$ *if* $a < 0$

The **roots or x-intercepts** are determined by equating y to zero and solving for x.

3.3 Sketch the graph of a parabola: You need:

- the y-intercept (here $x = 0$)
- the x-intercepts (here $y = 0$)
- the axis of symmetry (here $x = -\dfrac{b}{2a}$ obtained by noting that at the turning point of

  $y = ax^2 + bx + c = 0$ we have $\dfrac{dy}{dx}$, so $2ax + b = 0$. That means $x = \dfrac{b}{2a.}$

- the maximum/ minimum value is obtained by substituting the axis of symmetry in the given equation.

Example:
Sketch the graph of $y = f(x) = x^2 - 5x - 6$

y-intercept
If $x = 0$, then $f(0) = -6$

x-intercepts
If $y = 0$, then $x^2 - 5x - 6 = 0$

$$(x - 6)(x + 1) = 0$$
$$x = 6 \text{ or } x = -1$$

The axis of symmetry

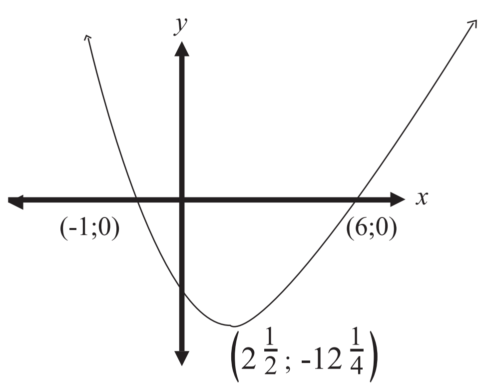
$$x = \frac{-b}{2a}, \text{ so } x = \frac{-5}{2(1)}$$

$$x = -\frac{5}{2}$$

Corresponding y-value (maximum/ minimum)

$$f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - \frac{5}{2} - 6$$

$$= 12\frac{1}{4}$$



Finding the equation of a given parabola:

- given the roots x_1 and x_2, use $y = a(x - x_1)(x - x_2)$
- given the turning point (p;q), use $y = a(x - p)^2 - q$
- given three points on the on the graph, with one being the intercept, substitute the other two points into $y = ax^2 + bx + c$ and solve the equations simultaneously. The y-value is given by the y-intercept simultaneous equations.
- Make x or y the subject of the formula in a simpler equation, such as a linear equation.
- Avoid solving for x or y if it has a co-efficient other than 1.
- Substitute for x or y in a more complex equation, such as a quadratic equation.
- Then solve the resulting equation.

• Substitute these values in either equation to get the corresponding values.
Examples

4. Solve for x and y:

$y + 2x = 2$ …….. (1)

$2x^2 + y^2 = 3yx$ ….. (2)

From (1) $y = 2-2x$ ….. *(3)*

Substitute (3) into (2)

$2x^2 + (2 - 2x)^2 = 3x\,(2 - 2x)$

$2x^2 + 4 - 8x + 4x^2 = 6x - 6x^2$

$12x^2 - 14x + 4 = 0$

$6x^2 - 7x + 2 = 0$

$(3x - 2)(2x - 1) = 0$

$x = \frac{2}{3}$ or $x = \frac{1}{2}$

Substitute into (3)

$y = 2 - 2\left(\frac{2}{3}\right)$ or $y = 2 - 2\left(\frac{1}{2}\right)$

$= \frac{2}{3}$ $\quad = 1$

5. Determine the equation of the parabola passing through (1;2) and (-2;-2) and cutting the y-axis -1.

Solution:

Using the y-intercept, we get $y = ax^2 + bx - 1$

(1;2) gives $2 = a + b - 1$……………..(1)

(-2;-2) gives $-2 = a(-2)^2 + b\,(-2) - 1$

$-2 = 4a - 2b - 1$…….(2)

From (1): $a = 3 - b$

Substitute into (2) : $-2 = 4(3-b) - 2b - 1$

$-2 = 12 - 6b - 1$

$b = -\frac{13}{6}$

$a = \frac{31}{6}$

6. Determine the equation of a graph with x-intercepts at -2 and 4 and with y-intercept at -6.

Solution:

$y = a(x - x_1)(x - x_2)$

$y = a(x + 2)(x - 4)$

The y-intercept is (0;-6)

So $-6 = a(0+2)(0-4)$

$-6 = -8a$

$a = \frac{3}{4}$

$y = \frac{3}{4}x^2 - \frac{3}{2}x - 6$

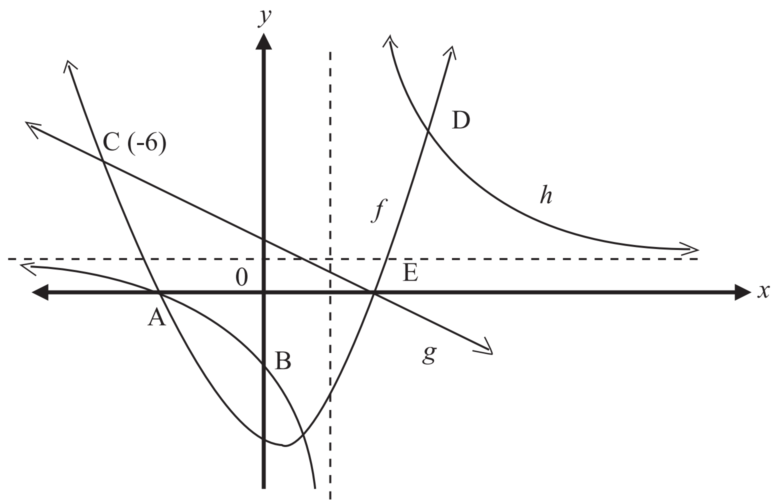
7. Determine the equation of a parabola with turning point (1;2) and passing through (2;-4).

Solution:
$y = a(x\text{-p})^2 + q$
$y = a(x-1)^2 + 2$ since (1;2) is the turning point
$-4 = a(2-1)^2 + 2$ since (2;-4) is on the parabola
$a = -6$
$y = -6x^2 + 12x - 4$

8. Sketched below are the graphs of: $g(x) = -2x + 8$, $f(x) = x^2 + k$ and

$$h(x) = \frac{6}{x-2} + 1$$

A and B are the x - and y - intercepts of h respectively, C $(-6\,;20)$ and E is the point of intersection of f and g.



8.1 Calculate the coordinates of A, B and E.
8.2 Show that the value of $k = -16$.
8.3 Determine the domain and the range of f.
8.4 Write down the values of x for which $g(x) - f(x) \geq 0$.
8.5 Determine the equation of the symmetry axis of h if the gradient is negative.
8.6 Write down the range of s, if $s(x) = f(x) + 2$.
8.7 Write down the range of t, if $t(x) = h(x) + 2$.

Solutions:
To answer the above questions, you need to identify all the functions, in order to apply the deductions indicated above.

At A, $y = 0$, so $\dfrac{6}{x-2} + 1 = 0$
$6 = -x + 2$
$4 = -x$
$x = -4$

Thus, A (-4;0)

At B, $x = 0$, so $y = \dfrac{6}{-2} + 1$
$y = -3 + 1$
$y = -2$

Thus, B (0;-2)

8.1 A and B are the x and y intercepts of g, respectively.
E is the x-intercept of the straight line and the parabola. It is easy and straight forward to use the equation of the straight line to get the coordinates of E.

At E, $y = 0$, then $0 = -2x + 8$
$2x = 8$
$x = 4$ Thus E (4;0)

8.2 C(-6; 20) is on f and g, substituting into

$y = x^2 + k \Rightarrow 20 = (-6)^2 + k$
$20 - 36 = k$
$k = -16$

8.3 Domain is $x \in \mathbb{R}$

Range is $y \geq -16; y \in \mathbb{R}$

8.4 These are values of x for which the graph of g and f intersect or f is below g.
It is from C (-6;20) and E (4;0)
That is $-6 \leq x \leq 4$

8.5 For negative gradient, $y = -(x - 2) + 1$
$y = -x + 2 + 1$
$y = -x + 3$

8.6 $+2$ implies the value of p is increased by 2.
The range of s is $y \geq -16 + 2$
$y \geq 14$

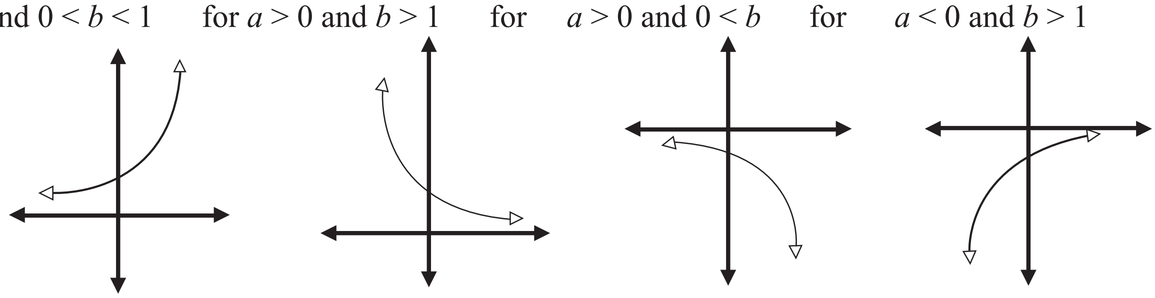
8.7 $+2$ implies the value of p is increased by 2.
The range of t is $y \neq 1 + 2; y \in \mathbb{R}$
$y \neq 3$; $y \in \mathbb{R}$

5.4 EXPONENTIAL

5.4.1 General representation or equation:

$y = ab^x$ *or* $y = ab^x + q$ *or* $y = ab^{x+p} + q$
The restriction is $b > 0; b \neq 1$

5.4.2 Important deductions
for $a < 0$ and $0 < b < 1$ for $a > 0$ and $b > 1$ for $a > 0$ and $0 < b$ for $a < 0$ and $b > 1$



• For $y = ab^x$, the asymptote is $y = 0$ and the y-intercept is $y = a$
• For $y = ab^{x+p} + q$, the asymptote is $y = q$ and y-intercept is $y = a + q$

• For $y = ab^{x+p} + q$, the asymptote is $y = q$ and y-intercept is $y = ab^{p} + q$

5.4.3 CONCEPTS AND SKILLS:

• Sketch the graph of the exponential functions defined by with correct shape.

• Interpret the graph, i.e. determine the following: intercepts with the axes, domain, range, equations of asymptotes, increasing/ decreasing intervals and reflection.

• Determine the equation of a given exponential graph.

5.4.4 PRIOR KNOWLEDGE:

• Solving exponential equations.
• Solving simultaneous equations (intersection points).
• Sketching exponential graphs done in Grade 10.
• Identifying increasing and decreasing exponential graphs.
• Determining the asymptotes of the exponential graph.
• Determining domain and range.
• Making deductions from the given sketch.

NB: The starting point must be the "mother function".

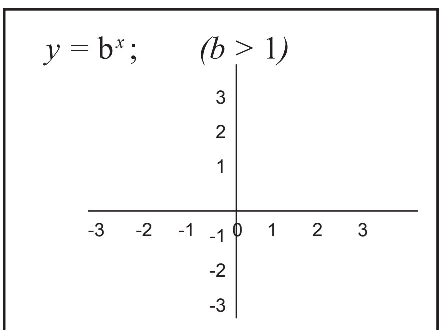
5.4.5 Definitions:

• Asymptotes: These are the lines that a graph 'tends towards', but doesn't reach.

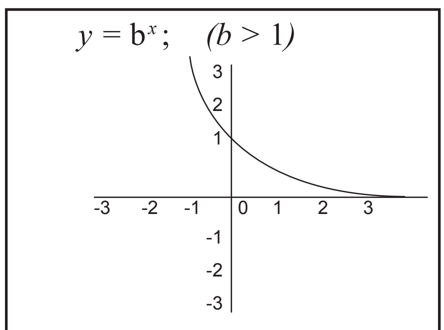
• A function is **increasing** if the values of y increase as x values increase.

• A function is **decreasing** if the values of y decrease as x values increase.

• **Domain:** These are x-values for which the graph is defined or a set of values assigned to the independent variables of a function.



Increasing function

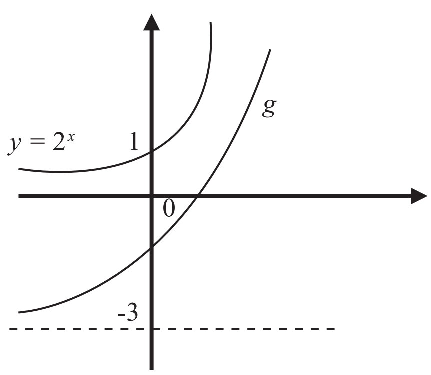


Decreasing function

• **Range:** A set of values that a function can take for all possible x-values.

• **Shapes**

EXAMPLE: When drawing an exponential graph, please begin by sketching the graph of $y = 2^{x}$; then draw a graph of $g(x) = 2^{x+1} - 3$



The **horizontal asymptote** is at $y = -3$
For x-intercept: Let $y = 0$; for y-intercept let $x = 0$

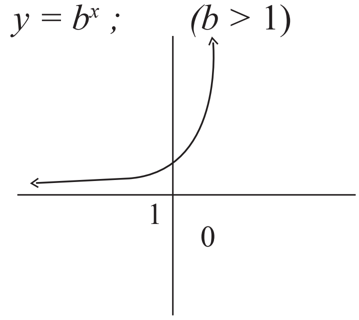
4.6 SKETCHING THE EXPONENTIAL EQUATION OF THE FORM:

Procedure for drawing graphs:

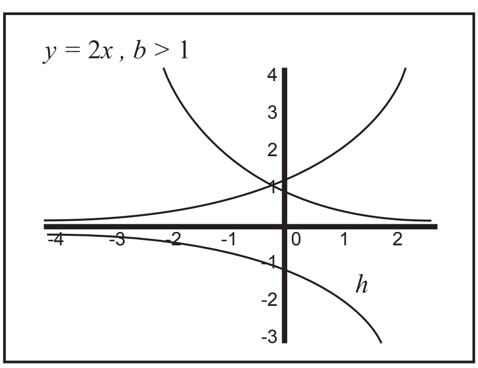
- Write down the asymptotes.
- Draw the asymptotes on the set of axes as dotted lines.
- Determine the x- intercept(s); let $y = 0$
- Determine the y- intercept(s); let $x = 0$
- Plot the points; then draw the graph using free-hand.

Example 1
Given the equation of graph of $f(x) = 2^x$:

(a) Write down the domain and range of *f*.
(b) Sketch the graph of f showing intercept(s) with axes.
(c) Write down the equation of the graph of g, i.e. the reflection of f in the y-axis.
(d) Write down the equation of graph h, i.e. the reflection of f in the x-axis.
(e) Write down the equation of the asymptote of g.
(f) Sketch the graphs of *f*, g and h on the same system of axes.
(g) Are *f*, *g* and h increasing or decreasing?



Solutions:
(a) Domain: $x \in \mathbb{R}$
(b) Range: $y > 0; y \in \mathbb{R}$
(c) $g(x) = 2^{-x}$ is the same as $g(x) = \left(\frac{1}{2}\right)^x$ which is a reflection of $f(x) = 2^x$ about the y-axis, since x has been replaced with $-x$.
(d) $h(x) = -f(x) = -2^x$
(e) $y = 0$
(f)



(g) *f* is an increasing function
g and *h* are decreasing functions

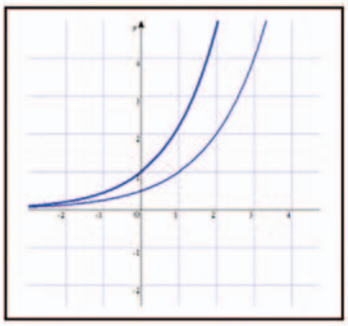
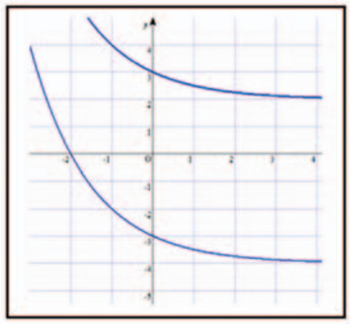
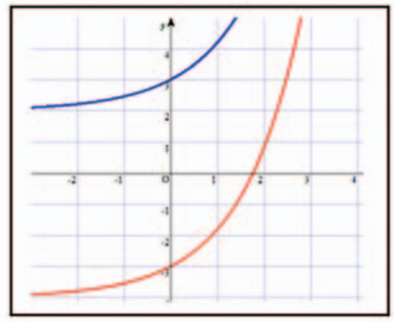
Sketch the graphs of the following on separate axes:

(a) $y = 2.2^x$ and $y = \frac{1}{2}.2^x$

(b) $y = \left(\frac{1}{2}\right)^x + 2$ and $y = \left(\frac{1}{2}\right)^x - 4$

(c) $y = {}^{2.2x} + 2$ and $y = 2.2^x - 4$

Draw a conclusion about the effect of a and q in the functions above.

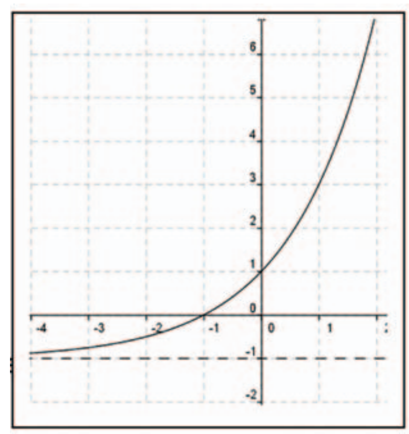




Solutions

If a becomes smaller, the graph opens or flattens

If q is positive, the graph moves upwards; if q is negative, the graph moves downwards

Sketch the graph of the form $y = ab^{x+p} + q; \quad (b > 0, b \neq 1)$



Example: Sketch the graph of the function f (x) = 2x+1 -1

Solution:

Step 1: y-intercept: $x = 0$

$y = 2^{0+1}$ **-1**

$= 1$

Step 2: Take x-value on the left of the y-intercept:

$y = 2^{-1+1} - 1$

$= 0$

Step 3: Take x- value on the right of the y-intercept:

$y = 2^{1+1} - 1$

$= 3$

Step 4: The equation of the horizontal asymptote: $y = -1$

Step 5: Draw the asymptote, plot the points and join them.

Given $f(x) = 3^{x-1} + 2$

(a) Write down the domain and the range of f.

(b) Sketch the graph of f, showing intercept(s) with axes.

(c) Write down the equation of the graph of g, i.e. the reflection of f in the y-axis.

(d) Write down the equation of graph h, i.e. the reflection of f in the x-axis.

(e) Write down the equation of the asymptote of g.

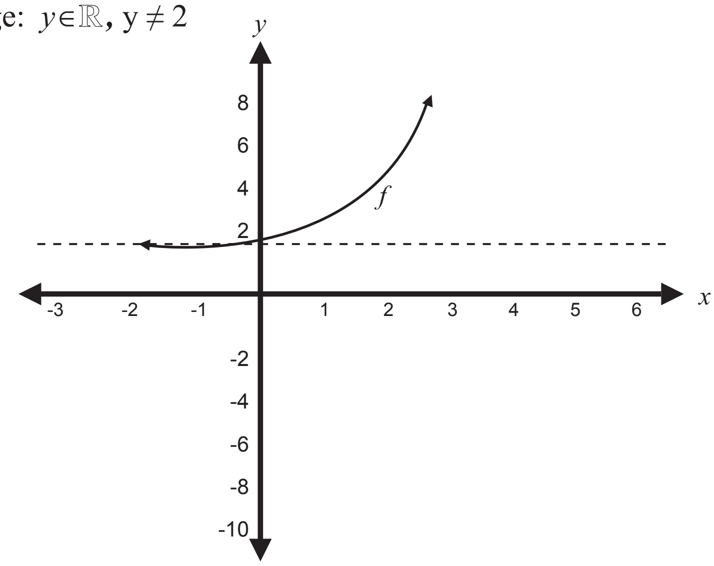
(f) Sketch the graphs of f, g and h on the same system of axis.

Solution:

(a) Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$, $y \neq 2$

(b)

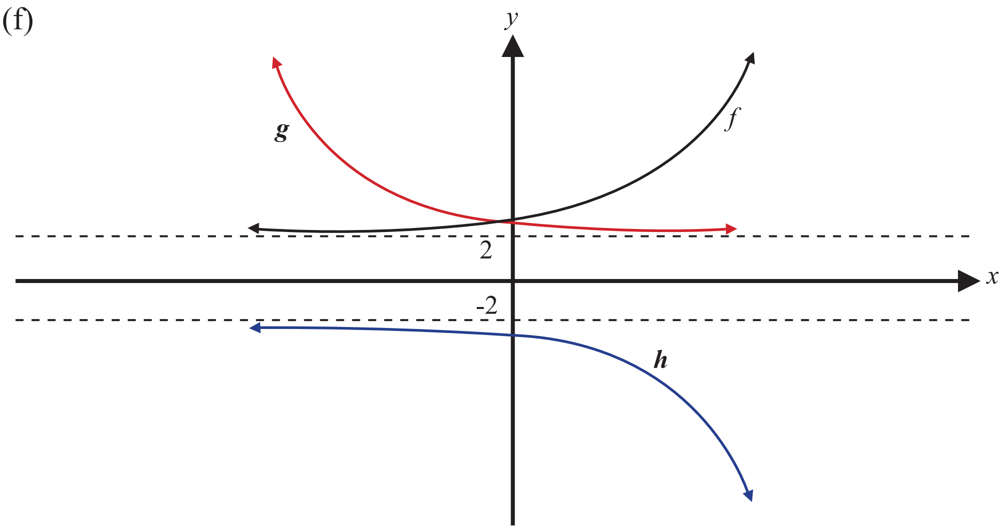


(c) $g(x) = 3^{-x-1} + 2$

(d) $h(x) = -3^{x-1} - 2$

(e) $x = 2$

(f)



CONSOLIDATION/ CONCLUSION

(a) Note that:

- For the graph of $y = ab^{x+p} + q$
- p affects the horizontal (right for $p < 0$ or left for $p > 0$) shift of the graph
- q affects the vertical (up for $q > 0$ or down for $q < 0$) movement of the graph

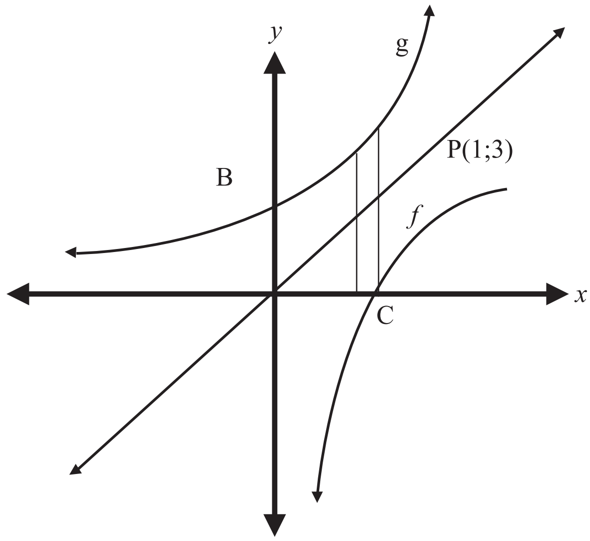
Activity 4.1

1. Complete the following table:

Function	Increasing or decreasing	Asymptote	x-intercept	Domain and range	Rough sketch
$y = 2^{x+1} - 2$					
$y = -2^{x-2} + 1$					
$y = 2.2^{x-3} - 4$					
$y = \left(\frac{1}{2}\right)^{x+1} + 2$					
$y = 2\left(\frac{1}{2}\right)^{x+2} - 2$					

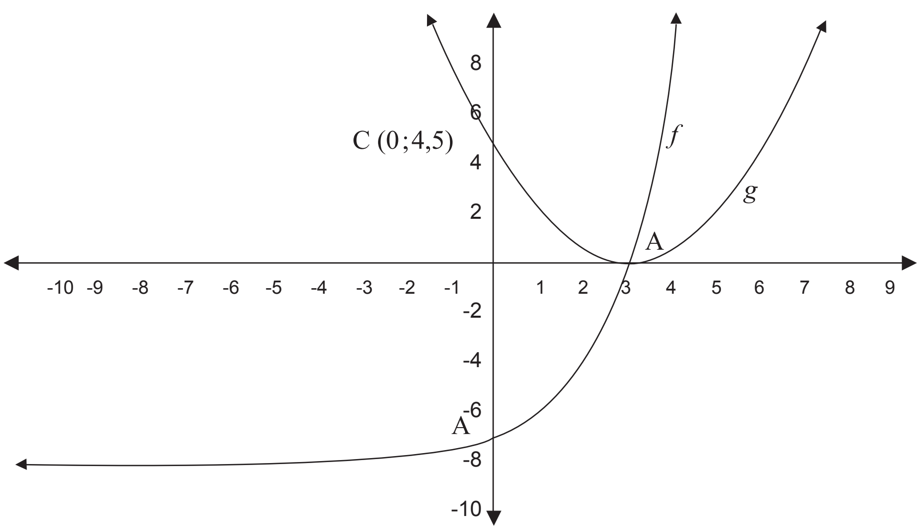
Draw each graph to confirm the properties and shapes written above.

2. In the figure, f is symmetrical to g about the line $y = x$. P(1 ; 3) is a point on g.



(a) Determine the value of a if $g(x) = a^x$.
(b) Determine the coordinates of B and C.
(c) Write down the equation of f.
(d) Determine the equation of h, if h is symmetrical to g about the y-axis.

3. The graphs of $f(x) = 2^x - 8$ and $ax^2 + bx + c$ are sketched below. B and C (0; 4,5) are the y-intercepts of the graphs of f and g, respectively. The two graphs intersect at A, which is the turning point of the graph of g and the x-intercept of the graphs of f and g.



(a) Determine the coordinates of A and B.
(b) Write down the equation of the asymptote of the graph of f.
(c) Determine the equation of h if $h(x) = f(2x) + 8$.
(d) Determine an equation of h^{-1} in the form $y = ...$
(e) Write down an equation of p, if p is the reflection of h-1 about the x-axis.

5.5 CUBIC GRAPHS

1. You need to know how to deal with cubic equations, in order to be able to deal with cubic graphs. Being able to differentiate is also helpful here.
2. There are two ways of finding the derivative, i.e. using the power rule and using first principles. First principles involves the definition $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

3. Differentiating by using the power rule (*If* $f(x) = ax^n$, then $f'(x) = anx^{n-1}$) is very useful when dealing with cubic graphs.
4. Don't forget, derivative notation $f'(x)$.
5. Simplify the expression first, removing any surds and quotients, etc.
6. The following simplification rule will prove helpful: $\frac{a+b}{c} = \frac{a+b}{c} + \frac{b}{c}$ and
7. It useful to know $\sqrt[3]{x^5} = x^{\frac{5}{3}}$

Examples

1. Use two methods to show that when you differentiate $y = x^2$ you get the same answer.

   Solution:

   Using the rule:

   $$\frac{dy}{dx} = 2x$$

   Using first principles:

   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   $$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

   $$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

   $$= \lim_{h \to 0} (2x + h)$$

   $$= 2x$$

Both methods yield the same answer.

2. Determine if $\frac{dy}{dx}$ *if* $y = (x + 1)(x - 2)$

   Solution:

   $y = x^2 + x - 2x - 2$

   $= x^2 - x - 2$

   $\frac{dy}{dx} = 2x - 1$

3. Compare the answers you get when using two methods to differentiate.

   $f(x) = -\frac{2}{x}$

4. Differentiate:

4.1 $f(x) = \dfrac{(x+2)^3}{\sqrt{x}}$

4.2 $y = \dfrac{x^2 - 1}{2x + 2}$

4.3 $y = \sqrt[3]{x^2} - \frac{1}{2}x$

Solutions:

4.1 $f(x) = \dfrac{x^3 + 4x^2 + 8x + 8}{x^{\frac{1}{2}}}$

$= x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + 8x^{-\frac{1}{2}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$

$= x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + \dfrac{4}{\sqrt{x}} - \dfrac{4}{\sqrt{x^3}}$

4.2 $y = \dfrac{x^2 - 1}{2x + 2}$

$= \dfrac{(x+1)(x-1)}{2(x+1)}$

$= \dfrac{x}{2} - \dfrac{1}{2}$

$\dfrac{dy}{dx} = \dfrac{1}{2}$

4.3 $y = x^{\frac{3}{2}} - \dfrac{1}{2}x$

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} - \dfrac{1}{2}$

The cubic function is important. You will be asked to either sketch the graph of a cubic function, or to determine its equation if the graph is given. Remember that, at a turning point, the derivative is zero. Note:

1. A cubic graph could be written as: y $= m(x - x_1)(x - x_2)(x - x_3)$ where $x_1; x_2; x_3$ are x-intercepts.
2. Another form of a cubic equation is: $= m(x - x_1)(x - x_2)^2$, where x_2 is a stationary point and also an x-intercept.

5. Given $f(x) = x^3 - 4x^2 - 11x + 30$

5.1 Use the fact that $f(2) = 0$ to write a factor of $f(x)$.

5.2 Calculate the coordinates of the intercepts of $f(x)$.

Solution:

$$\begin{aligned} f(x) &= x^3 - 4x^2 - 11x + 30 \\ &= (x - 2)(x^2 - 2x - 15) \\ &= (x - 2)(x + 3)(x - 5) \end{aligned}$$

$f(x) = 0$

$(x + 3)(x - 2)(x - 5) = 0$

$x = -3$ or $x = 2$ or $x = 5$

x-intercepts $(-3,0)$; $(2,0)$ and $(5,0)$

5.3 Calculate the coordinates of the stationary point of f.

Solution:

$f(x) = x^3 - 4x^2 - 11x + 30$

$f(x) = 3x^2 - 8x - 11$

At turning points $f'(x) = 0$

$(x + 1)(3x - 11) = 0$

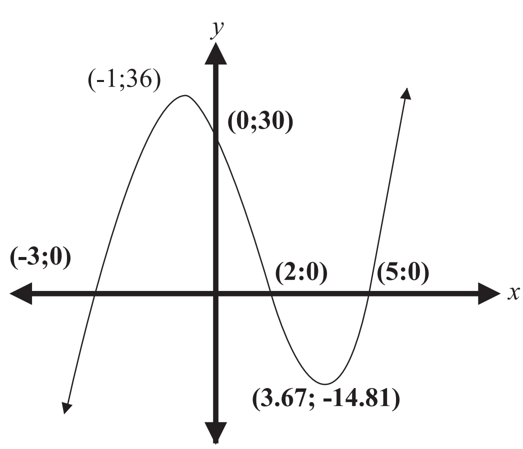
$x = -1$ or $x = -\dfrac{11}{3}$

if $x = -1$, then $y = 36$ and if $x = -\dfrac{11}{3}$ then $y = -\dfrac{400}{27}$

TP are $(-1;36)$ and $\left(\dfrac{11}{3}; -14{,}81\right)$

5.4 Sketch the curve of f. Show all intercepts with the axes.

Solution:



5.5 For which values of x will $f(x) < 0$?

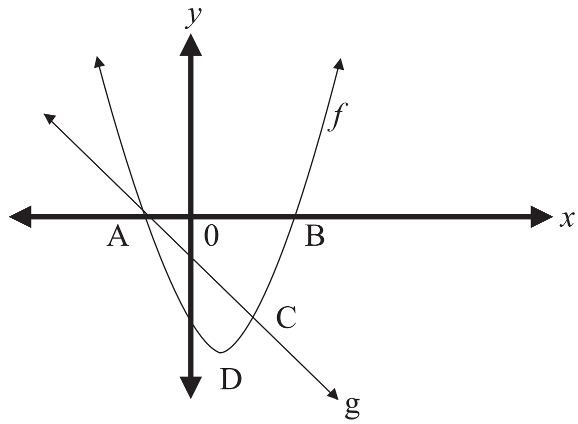
Solution:

$f(x) < 0$ if $-1 < x < 3{,}67$

QUESTIONS FROM PAST EXAMINATION PAPERS

QUESTION 1

1.1 Sketched below are the functions: $f(x) = 2x^2 - 6x - 20$ *and* $g(x) = -2x + k$.



Determine:

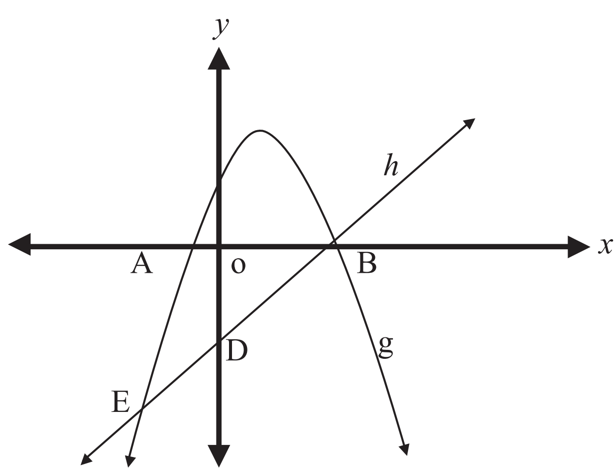
1.1.1 The coordinates of turning point D.
1.1.2 The coordinates of A and B.
1.1.3 The value of k .
1.1.4 The values of p, if $2x^2 - 6x + p = 0$ has no real roots.
1.1.5 For which values of x is $f(x).g(x) \leq 0$.
1.1.6 The value of t if $y = -2x + t$ is a tangent to f.

1.2 Consider the following two functions: $p(x) = x^2 + 1$ and $r(x) = x^2 + 2x$.

1.2.1 How will you shift p to become the function r ?
1.2.2 Write down the range of p.

QUESTION 2

The sketch below shows the graphs of $g(x) = - x^2 + 2x + 3$. and $h(x) = ax + q$. The graphs intersect at B and E. The graph of g intersects the x – axis at A and B and has a turning point at C. The graph of h intersects the y – axis at D. The length of CD is 6 units.



2.1 Determine the coordinates of B and C.
2.2 Write down the coordinates of D.
2.3 Write down the values of a and q.
2.4 Determine the coordinates of E.

QUESTION 3

The function f is given by the equation $y = - x^2 \;\; -2x + 8$

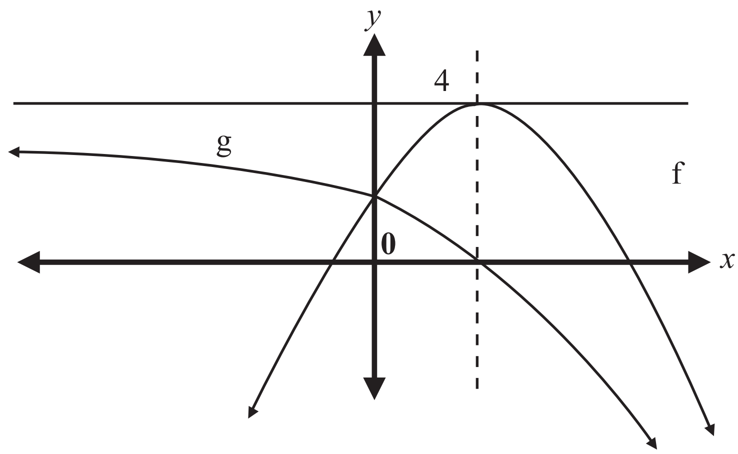
3.1 Determine all the intercepts of $f(x)$ with the axes.

3.2 Determine the turning point of $f(x)$.

3.3 Determine the value of c if the graph $g(x) = - 4x + c$ is a tangent to the graph of $f(x)$.

QUESTION 4

The graphs of the functions $g(x) = -1.2^{(x+p)} + q$ and $f(x = ax^2 + bx + c)$ are drawn below.



4.1 Show that $p = 1$ and $q = 4$.

4.2 Determine the y- intercept of $g(x)$.

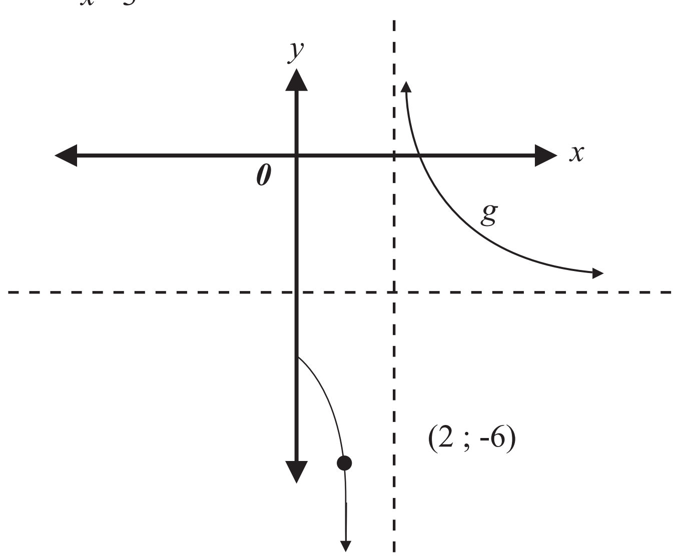
4.3 Determine the equation of $f(x)$.

4.4 Write down the range of $g(x)$.

4.5 Explain how increasing the value of q will change the graph of $g(x)$.

QUESTION 5

Below is the graph of $g : x \rightarrow \dfrac{2}{x - 3} + q$, with a point $(2; - 6)$ on the graph.



5.1 Write down the equations of the asymptotes of g.

5.2 Determine the domain of the graph of g.

5.3 Determine the equation of $f(x) = -2\, g(x + 2)$.

QUESTION 6

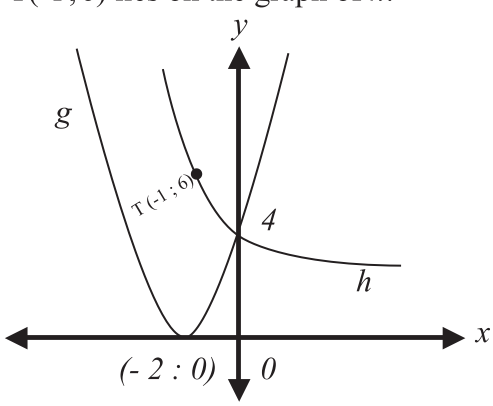
The equation of a function f is given by

$$f(x) = \frac{-3}{x - 2} + 1$$

6.1 Determine the asymptotes of $f(x)$.
6.2 Determine the x and y intercepts.
6.3 Draw a neat sketch of f on the given diagram sheet.
6.4 Write down the asymptotes of $k(x)$, *if* $k(x)$ is a function where $f(x)$ is moved 5 units to the left and 1 unit down.

QUESTION 7

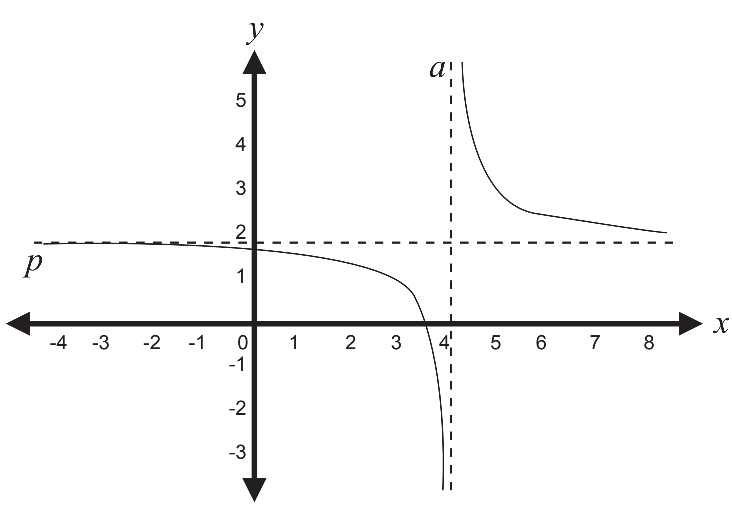
In the diagram is the graphs of: $h(x) = px + q$ and $g(x) = ax^2 + bx + c$, f touches the x axis at (-2 ; 0); h and g intersect at (0;4); T(-1;6) lies on the graph of h.



7.1 Determine the equation of g.
7.2 Determine the value of p and q.
7.3 For what values of x is $h(x) < g(x)$?
7.4 Describe the transformation from g to p if $p(x) = x^2$

QUESTION 8

The diagram below shows the graph of: $f(x) = \frac{a}{x - p} + q$ T (5;3) is a point on f.



8.1 Determine the values of p, q and a.
8.2 Write down the equation of $p(x)$ if $p(x) = f(x+1) - 2$.
8.3 If the new graph p(x) is reflected across the line with equation $y = -x + c$, then the new graph will be exactly $y = f(x)$. Determine the value of c.

QUESTION 9

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

9.1 Determine the equation of f^{-1} in the form $f^{-1}(x)$…
9.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph for each of f and f^{-1}.
9.3 Is f^{-1} a function? Give a reason for your answer.

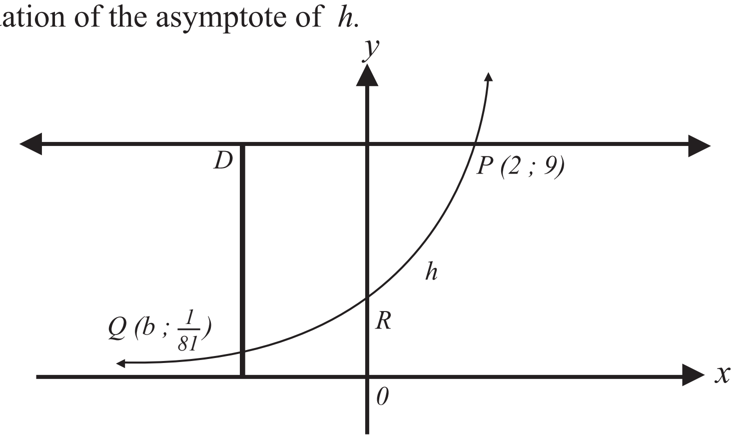
QUESTION 10

Given: $f(x) = 2^{-x} + 1$

10.1 Determine the coordinates of the y-intercept of f.
10.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes.
10.3 Calculate the average gradient of f between the points on the graph, where $x = -2$ and $x = 1$.
10.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h.

QUESTION 11

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the y-intercept of h.
The points P(2 ; 9) and $q\left(b; \frac{1}{81}\right)$ lie on h.
11.1 Write down the equation of the asymptote of h.



11.2 Determine the coordinates of R.
11.3 Calculate the value of a.
11.4 D is a point such that DQ || y-axis and DP || x-axis. Calculate the length of DP.
11.5 Determine the values of k for which the equation $h(x+2) + k = 0$ will have a root that is less than -6.

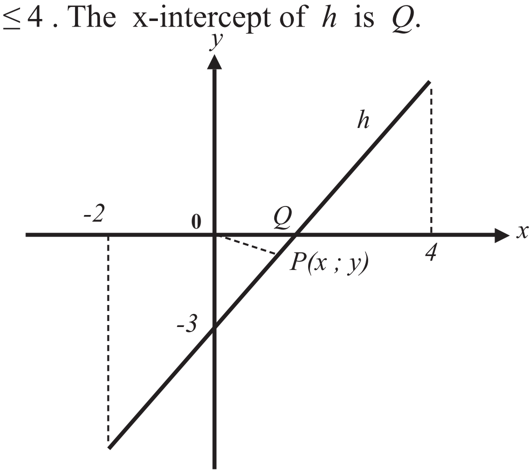
QUESTION 12

Given: $f(x) = -x + 3$ *and* $g(x) = log_2 x$

12.1 On the same set of axes, sketch the graphs of f and g, clearly showing ALL intercepts with the axes.
12.2 Write down the equation of $g^{-1}(x)$ the inverse of g, in the form $y = ...$
12.3 Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3 - x) = x$
12.4 Write down the solution of $\log_2 (3 - x) = x$.

QUESTION 13

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x-intercept of h is Q.

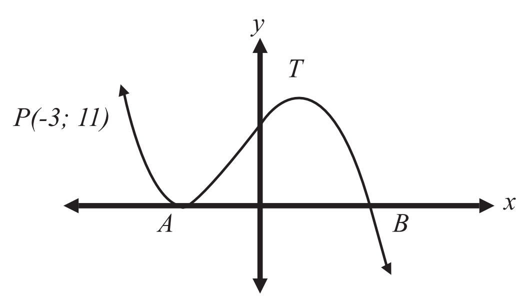


14.1 Determine the coordinates of Q.
14.2 Write down the domain of h^{-1}.
14.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points.
14.4 For which value(s) of x will $h(x) = h^{-1}(x)$?

14.5 $P(x ; y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP.
14.5 Given: $h(x) = f'(x)$,where f is a function defined for $-2 \leq x \leq 4$.
14.6.1 Explain why f has a local minimum.
14.6.2 Write down the value of the maximum gradient of the tangent to the graph of f.

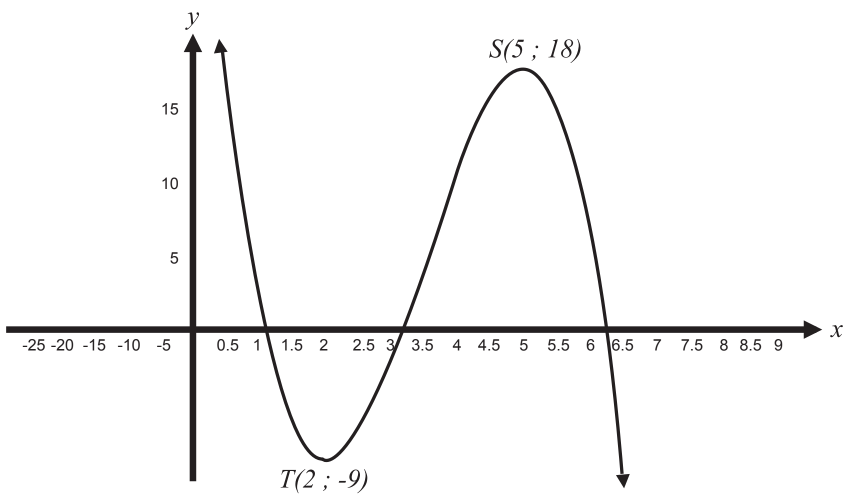
QUESTION 16

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$. A and T are the turning points of g. A and B are the x-intercepts of g. P(–3; 11) is a point on the graph.



16.1 Determine the length of AB.
16.2 Determine the x-coordinate of T.
16.3 Determine the equation of the tangent to g at P(–3; 11) in the form y = …
16.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots.
16.5 Determine the x-coordinate of the point of inflection.

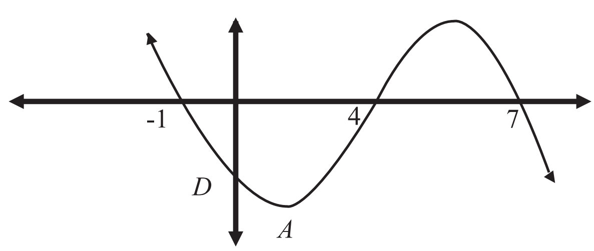
QUESTION 17



The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched above.
The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).
17.1 Show that $a = 21$, $b = -60$ and $c = 43$.
17.2 Determine the equation of the tangent to the graph of f at $x = 1$.
17.3 Determine the x-value at which the graph of f has a point of inflection.

QUESTION 18

The sketch below shows the graph of $f(x) = -x^3 + 10x^2 - 17x + d$. The x-intercepts of f are (-1;0), (4;0) and (7;0). A and B are the turning points of f ; D is the y-intercept of f. The sketch is not drawn to scale.
18.1 Write down the value of D.



18.2 Determine the coordinates of A and D.
18.3 Determine the value of x, where the concavity of f changes.
18.4 Determine the coordinates of the point on f with a maximum gradient.

6. Check your answers

- Indicate answers / responses to the questions and activities in this section.
- Please ensure accurate correlation to the activity in the previous section;
- Indicate mark allocation (use ticks ✔) where required, but more importantly, explain how marks are allocated in the examination.

7. Message to Grade 12 learners from the Writers

Mathematics can be fun as it requires you to pull all the pieces you learnt in the lower grades to answer the Grade 12 examination questions. If you have skipped one grade before Grade 12, I had left a void to ground the floor.

Please ensure that you know all axioms and corollaries (all rules) to answer questions. Revise at least three sets of previous Mathematics question papers before you sit the final examinations.

Write one paper in 3 hours and mark your script on your own, using the memorandum, in order to gauge if you are ready to sit the final paper. Memoranda for all previous DBE examinations are available on the DBE's website.

I assure you, this year's final paper will be similar to the previous years' papers in terms of format and style.

Eat a small amount of food prior to the exam, to ensure your cerebrum (brain) works promptly and effectively. Digestion occupies the functioning of your brain.

Blast the final paper.

8. Thank you / Acknowledgements

We hope the explanations are well received in preparation for your final examinations. Mr Leonard Gumani Mudau, Mrs Nonhlanhla Rachel Mthembu, Ms Thandi Mgwenya and Mr Percy Steven Tebeila wish you well.

MATHEMATICS

FUNCTIONS

GRADE 12

Department of Basic Education
222 Struben Street, Pretoria, 0001
Private Bag X895, Pretoria, 0001, South Africa
Tel: (012) 357 3000   Fax: (012) 323 0601

Hotline: 0800 202 933

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